XSP Technology

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NOTES ON: ITEMS, SETS, NAMES, TUPLES, & KLASSES

Given an *element* $\mathbf{x}$ and a *scope* $\mathbf{y}$, $\mathbf{x} \in_{\mathbf{y}} \mathbf{Q}$ asserts that $\mathbf{x}$ is a $\mathbf{y}$-member of set $\mathbf{Q}$ and $\{\mathbf{x}^{\mathbf{y}}\} \subseteq \mathbf{Q}$. *Items* are to be considered as identifiable entities that can be distinguished, using the concept of set membership, as being of one of two types. There are those items that have members and there are those items that do not have members. Formally and in conjunction with the axioms (http://xsp.xegesis.org/X_axioms.pdf) of extended set theory (XST).

'$\mathbf{x}$' is an item having members **IFF** $\mathcal{X}(\mathbf{x})$, where $\mathcal{X}(\mathbf{x}) \iff (\exists\, a, s)(a \in_s \mathbf{x})$, and

'$\mathbf{x}$' is an item with no members **IFF** $\neg\mathcal{X}(\mathbf{x})$, where $\neg\mathcal{X}(\mathbf{x}) \iff (\forall\, a, s)(a \notin_s \mathbf{x})$.

Every item has a *successor* which is defined by: $!\mathbf{x} \;=\; \mathbf{x} \cup \{\mathbf{x}^{\mathbf{x}}\}$.

Items having members qualify as *sets*. Some items, which do not qualify as sets, do qualify as *names*. Formally, '$\mathbf{x}$' is a name for a set **IFF** $\mathcal{X}(\eta(\mathbf{x}))$ & $\neg\mathcal{X}(\mathbf{x})$. *Natural Names* can be expressed using the item successor definition, above, by: $\eta(\mathbf{1}) = \{\emptyset^{\emptyset}\}$, $\eta(\mathbf{2}) = !\eta(\mathbf{1}) = !\{\emptyset^{\emptyset}\} = \left\{ \emptyset^{\emptyset},\ \{\emptyset^{\emptyset}\}^{\{\emptyset^{\emptyset}\}} \right\}$, $\eta(\mathbf{3}) = !\eta(\mathbf{2})$

The set of all Natural names, $\mathcal{N}$, can be defined by:

1) $(\forall\mathbf{x},\mathbf{y})(\ \mathbf{x} \in_{\mathbf{y}} \mathcal{N} \ \rightarrow\ \mathbf{x} = \mathbf{y}\ \&\ \eta(\mathbf{x}) \neq \emptyset\ )$, and

2) $(\forall\mathbf{x})(\ \mathbf{x} \in_{\mathbf{x}} \mathcal{N} \ \rightarrow\ \eta(\mathbf{x}) = \{\emptyset^{\emptyset}\}$ or $(\exists\mathbf{y})(\ \mathbf{y} \in_{\mathbf{y}} \mathcal{N}\ \&\ \eta(\mathbf{x}) = !\eta(\mathbf{y}))\ )$.

It will be convenient to define variable names, '$n+1$' and '$1+n$', by: $\eta(\mathbf{n}+1) \;=\; \eta(1+\mathbf{n}) \;=\; !\eta(\mathbf{n})$, giving, for example: $\eta(4) \;=\; \eta(\mathbf{3}+1) \;=\; \eta(1+\mathbf{3}) \;=\; !\eta(\mathbf{3})$. Thus: $\mathbf{1} \in_{\mathbf{1}} \mathcal{N}$, $\mathbf{2} \in_{\mathbf{2}} \mathcal{N}$, $\mathbf{3} \in_{\mathbf{3}} \mathcal{N}$, and $(\mathbf{n}+1) \in_{(\mathbf{n}+1)} \mathcal{N}$ (for all $\mathbf{n} \in_{\mathbf{n}} \mathcal{N}$), where, $\mathcal{N} = \{\ 1^1, 2^2, .., n^n, ...\}$ and $\mathcal{N}(n) = \{\ 1^1, 2^2, .., n^n\ \}$ as Natural names up to and including 'n', with $\mathcal{N}(n) = \{\ \mathbf{x}^{\mathbf{x}} :\ \mathbf{x} \in_{\mathbf{x}} \mathcal{N}\ \ \&\ \ \eta(\mathbf{x}) \subseteq \eta(n)\ \}$. Define $\mathcal{N}_{\emptyset}(n) = \mathcal{N}(n) \cup \{\emptyset^{\emptyset}\}$.

Set names can be equated and ordered by mimicking the inclusion relationship (if any) that holds between the sets being named. Two names, '$\mathbf{x}$' and '$\mathbf{y}$' are considered equal, $\mathbf{x}=\mathbf{y}$, when their respective sets are equal, $\eta(\mathbf{x}) = \eta(\mathbf{y})$. Name '$\mathbf{x}$' is considered less than name '$\mathbf{y}$', $\mathbf{x} < \mathbf{y}$, when $\eta(\mathbf{x}) \subset \eta(\mathbf{y})$. This ordering relation imposed on natural names can be used to provide a natural definition for an n-tuple, as follows: $< x_1, x_2, .., x_n >= \{x_1^1, x_2^2, .., x_n^n\}$. This is a radical departure from the usual nested set definition of n-tuple and it will not qualify as a Classical set.

Sets that can be defined under a Classical set theory, CST, will be denoted by $Cst(\mathbf{A})$ being true. This requires that all scopes of elements of $\mathbf{A}$, at all nested levels of $\mathbf{A}$ are $\emptyset$. This can be formally expressed by:

$$Cst(\mathbf{A}) \iff (\forall x, y)(x \in_y \mathbf{A} \rightarrow y = \emptyset\ \ \&\ \ (\neg\mathcal{X}(x)\ or\ (\ \mathcal{X}(x)\ \&\ Cst(x)\ ))).$$

In general, collections of sets can be classified by their immediate and nested scope content. For every set there exists a characteristic minimal scope set defined by:

$$\mathcal{S}_*(A) \;=\; \left\{ s^s :\ (\exists x)(x \in_s A)\ \text{ or }\ (\exists B, v)\left(\ B \in_v A\ \ \&\ \ s \in_s \mathcal{S}_*(B)\ \right)\right\}.$$

For all CST sets $Cst(\mathbf{A}) \iff \mathcal{S}_*(A) = \{\emptyset^{\emptyset}\}$, but since $\mathcal{S}_*(< x_1, x_2, .., x_n >) = \{\emptyset^{\emptyset}, 1^1, 2^2, .., n^n\} = \mathcal{N}_{\emptyset}(n)$, n-tuples can not belong to any CST set, nor even belong to the set of all CST sets. Let $\textsf{SET} \;=\; \{\ x^{\eta(\mathbf{1})} :\ Cst(x)\ \}$, then $\mathcal{S}_*(\textsf{SET}) = \{\emptyset^{\emptyset}, \eta(\mathbf{1})^{\eta(\mathbf{1})}\} = \eta(\mathbf{2})$. In CST the *Power Set* of a CST set $\mathbf{Q}$, $\mathcal{P}(\mathbf{Q})$, or set of all subsets of $\mathbf{Q}$, is also a CST set, but this is not so with the following definition: $\mathcal{P}(\mathbf{Q}) \;=\; \{\mathbf{A}^s :\ \mathbf{A} \subseteq \mathbf{Q}\ \ \&\ \ s = \mathcal{S}_*(\mathbf{Q})\}$. This extended definition does however preserve a 1-1 mapping to the CST definition and in addition allows the set of all CST subsets to be defined by $\mathcal{P}(\textsf{SET})$. This extended definition also provides a mechanism to construct *larger* sets.

Certain sets are so *large* as to deserve special consideration. $\mathcal{K}$lasses are sets defined by a dominant relationship between nested levels of scope sets. A α-$\mathcal{K}$lass is defined by: $\mathcal{K}_\alpha \;=\; \{\mathbf{x}^\alpha :\ \mathcal{X}(\mathbf{x})\ \ \&\ \ \alpha = \mathcal{S}_*(\mathbf{x})\ \} \cup \{\emptyset^\alpha\}$. Note that for all α, $\mathcal{S}_*(\mathcal{K}_\alpha) = !\alpha$, that $\textsf{SET} = \mathcal{K}_{\eta(\mathbf{1})}$, and that $\mathcal{P}(\textsf{SET}) \subset \mathcal{K}_{\eta(\mathbf{2})}$. Like other sets $\mathcal{K}$lasses can also be qualified by conditional properties, as in: $\mathcal{K}_\alpha(\mathbf{P}) = \{\mathbf{x}^\alpha :\ \mathcal{X}(\mathbf{x})\ \ \&\ \ \mathbf{P}(\mathbf{x})\ \ \&\ \ \alpha = \mathcal{S}_*(\mathbf{x})\ \}$. $\mathcal{K}$lasses are intended to provide a stratification of sets that allows for defining *large sets* of *large sets* without fear of bumping into antinomies.

For arbitrary large sets, $\mathbf{A}_i$, $< \mathbf{A}_1, \mathbf{A}_2, .., \mathbf{A}_n >$ may not always be well defined. It can easily be shown that when each $\mathbf{A}_i$ is a subset of some $\mathcal{K}$lass, $< \mathbf{A}_1, \mathbf{A}_2, .., \mathbf{A}_n >$ is always well defined. The following are all well-defined: $<\textsf{SET},\ !\textsf{SET}>$, $<\mathcal{K}_\sigma, \mathcal{K}_\tau, \mathcal{K}_\gamma>$, $<<\textsf{SET},\ !\textsf{SET}>, \mathcal{P}(\mathcal{P}(\textsf{SET}))>$, and are many other bizarre constructions.