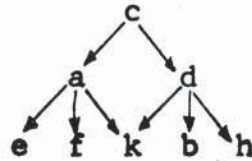


BINARY RELATIONS

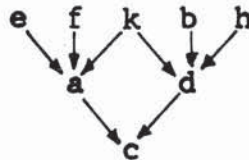
$$A = \{ \langle c, a \rangle, \langle c, d \rangle, \langle a, e \rangle, \langle a, f \rangle, \langle a, k \rangle, \langle d, k \rangle, \langle d, b \rangle, \langle d, h \rangle \}$$



$$\text{CARD}(A) = 8$$

CONVERSE of A: $CV(A,C)$

$$C = \{ \langle a, c \rangle, \langle d, c \rangle, \langle e, a \rangle, \langle f, a \rangle, \langle k, a \rangle, \langle k, d \rangle, \langle b, d \rangle, \langle h, d \rangle \}$$



1. Find BEGIN and END points.

$$\begin{aligned} \text{DM}(A,B) &= \text{RG}(C,B) & B &= \{a, c, d\} \\ \text{RG}(A,E) &= \text{DM}(C,E) & E &= \{a, b, d, e, f, h, k\} \end{aligned}$$

2. Find FIRST and LAST points.

$$\begin{aligned} \text{RL}(B,E,F) & & F &= \{c\} \\ \text{RL}(E,B,L) & & L &= \{b, e, f, h, k\} \end{aligned}$$

3. Given points P, find successor points S.

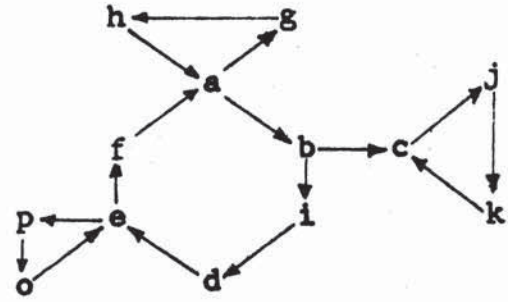
$$\begin{aligned} \text{IM}(A,P,S) & \text{ If } P = \{c\} & \text{ then } S &= \{a, d\} \\ & \text{ If } P = \{a\} & \text{ then } S &= \{e, f, k\} \end{aligned}$$

EXECUTION TIMES IN SECONDS ON IBM 360/67

#A=(2)		DOMAIN	CONVERSE	1 IMAGE	10%
(2)	100	0.004	0.041	.0006	.004
(2)	500	0.019	0.302	.0006	.027
(2)	1000	0.035	0.685	.0007	.069
(2)	5000	0.148	5.142	.0013	.769

THE RINGS

$A = \{ \langle i,d \rangle, \langle d,e \rangle, \langle e,f \rangle, \langle f,a \rangle, \langle a,b \rangle, \langle b,i \rangle, \langle a,g \rangle, \langle g,h \rangle, \langle h,a \rangle, \langle p,o \rangle, \langle o,e \rangle, \langle e,p \rangle, \langle c,j \rangle, \langle j,k \rangle, \langle k,c \rangle, \langle b,c \rangle \}$



CARD(A) = 16

Given points B, find all reachable points C.

```

#LIST REACH
> 1 SUBROUTINE REACH(A,B,C,T)
> 2 INTEGER A(1),B(1),C(1),T(1)
> 3 CALL UN(B,B,C)
> 4 100 CALL IM(A,B,T)
> 5 IF(CARD(T).EQ.0) RETURN
> 6 CALL RL(T,C,B)
> 7 IF(CARD(B).EQ.0) RETURN
> 8 CALL UN(C,B,C)
> 9 GO TO 100
> 10 END
  
```

ACTUAL RUNS

CARD(A) = 1000 gave CARD(C) = 12
 CARD(B) = 1 cpu-sec = 0.0207

CARD(A) = 5000 gave CARD(C) = 992
 CARD(B) = 1 cpu-sec = 2.48

OPERATIONS EXTENDED TO N-TUPLES

$$A = \left\{ \begin{array}{l} \langle p, q, r, s, t \rangle, \\ \langle k, z, m, n, o \rangle, \\ \langle a, b, c, d, e \rangle, \\ \langle u, v, w, x, y \rangle, \\ \langle f, g, h, i, j \rangle \end{array} \right\}$$

I-TH DOMAIN of A

$$\begin{array}{ll} \text{DM}(1, A, C) & C = \{a, f, k, p, u\} \\ \text{DM}(3, A, C) & C = \{c, h, m, r, w\} \end{array}$$

I-TH RESTRICTION of A to B

$$\text{Let } B = \{z, s\} \text{ then for } \text{RS}(2, A, B, C)$$

$$C = \{\langle k, z, m, n, o \rangle\}$$

and

$$\text{RS}(4, A, B, C)$$

$$C = \{\langle p, q, r, s, t \rangle\}$$

$$\text{LET } A = \{\langle \text{name}, \text{mother}, \text{father}, \text{spouse}, \text{sex} \rangle\}$$

GIVEN x , find all sisters of x .

$$\text{let } X = \{x\} \text{ and } W = \{\text{female}\}$$

$$\text{RS}(1, A, X, B) \quad B = \{\text{n-tuples with } x \text{ in position 1}\}$$

$$\text{DM}(2, B, M) \quad M = \{x\text{'s mother, } m\}$$

$$\text{DM}(3, B, F) \quad F = \{x\text{'s father, } f\}$$

$$\text{RS}(2, A, M, C) \quad C = \{\text{n-tuples with } m \text{ in position 2}\}$$

$$\text{RS}(3, A, F, D) \quad D = \{\text{n-tuples with } f \text{ in position 3}\}$$

$$\text{IN}(C, D, G) \quad G = \{\text{intersection of } D \text{ and } C\}$$

$$\text{RS}(5, G, W, H) \quad H = \{\text{n-tuples of } G, \text{ female in 5 pos.}\}$$

$$\text{RL}(\text{DM}(1, H), X, S) \quad S = \{\text{sisters of } x\}$$