

AXIOMATIC EXTENDED SET THEORY

A Focus On Extended Functions

Abstract

The principal intent of this paper is to extend the axioms and operations of set theory to accommodate a richer variety of functions than are currently provided by set theory, λ -calculus or category theory. This paper supplants an earlier work, initially presented at the IFIP Congress in 1968 [Ch68], that addressed extending functions, within the confines of existing axioms of set theory, through a 'reconstituted definition of relation'.

This current paper differs from the previous in two respects, though all the conclusions of the former are preserved. Instead of relying on the existing axioms of classical set theory, a new collection of axioms for extended set theory has been developed that now supports arbitrarily 'large sets of sets', including the set of all classical sets. The second difference is that the concept of an *Rship* is introduced to provide a richer definition for 'function', 'image', 'product' and 'composition'.

INTRODUCTION: For those whose work is adequately served by a definition for function provided by set theory, λ -calculus or category theory, the following material may be of little interest.

This paper is intended for those whose work could benefit from: a non-ambiguous definition of n -ary function; a non-ambiguous definition of n -th Cartesian product; a rigorous definition of ' n -to- m ' functions (e.g. $\sqrt[n]{x_1 \times x_2 \times \cdots \times x_n}$); a definition of 'Image' supporting ' n -to- m ' functions and ' $f(f) \neq \emptyset$ '; a definition of 'Composition' supporting multiple interacting n -ary functions; a definition of 'pair' that behaves in a 'more reasonable' fashion than does the Kuratowski definition (which gives: $\langle a, b \rangle \cap \langle a, x \rangle = \langle a, a \rangle$); a non-ambiguous definition for n -tuples as specific sets and not as a canonical choice from an equivalence class of binary codifications; and lastly, a single definition of function that supports an ascending spectrum of mappings involving 'sets-of-sets', 'classes-of-sets' and 'classes-of-classes'.

Motivation for extending the foundation of set theory is provided by an informal presentation of modeling difficulties arising from representations that allow *non-functional abstractions*, where current set theories are prone to fostering *non-functional abstractions*, how these *non-functional abstractions* can be removed (when necessary) from set theory, and how such an extended set theory can provide a closed model of systems that required 'intervention-decisions' when using an existing set theory (e.g. resolution of an instance blanketed by an isomorphism such as the specific membership for a 5-tuple or an interpretation of $\sqrt{\sqrt{16}}$).

The ultimate objective of this paper is to present an axiomatic foundation that preserves support for existing set theories while also providing support for *extended functions*.