XST NOTES: Tuplesets, Tagged-Sets, Application, & Etc.

Following is as succinct as possible an introduction to *Tuplesets* and their properties. A familiarity with XST concepts is assumed. Only a brief collection of XST definitions necessary to support Tupleset definitions will be given. The idea presented here is that, armed with a well-behaved definition for n-tuple (unavailable in CST, but well-defined in XST), a definition for *application* can be developed prior to defining the concept of *function* and that such a definition can support the concept of *self-application* when applied to specifically defined sets. (The supplement contains a preview of definitions for *functions* and *Kategories*.)

1 PRELIMINARY DEFINITIONS

Extended sets are collections defined by a qualified membership condition. When the qualification is *null* the extended set membership condition is equivalent to a Classical set membership condition.

Definition 1.1 *Set: Given the null set,* $\emptyset$*, and the extended membership predicate,* $\in$*, then:*

$$Y \text{ is a set} \longleftrightarrow (\exists x, s)(\, x \in_s Y \,) \text{ or } Y = \emptyset.$$

Notationally, XST sets are written as in CST except with subscripts on *element of*, $\in_s$, in conditional statements, and with superscripts on elements in extensional descriptions, $\{\, x^a, y^b, z^c \,\}$.

Definition 1.2 *Membership Convention:* $x \in Y \;\leftrightarrow\; (\exists s)(x \in_s Y)$,

Definition 1.3 *Scope Set:* $\mathcal{S}(\mathbf{A}) \;=\; \{y^y\!: \; (\exists x)(\, x \in_y \mathbf{A} \,)\}$.

Definition 1.4 *Element Set:* $\mathcal{E}(\mathbf{A}) = \{\, x^x\!: \; (\exists y)(\, x \in_y \mathbf{A})\}$.

Definition 1.5 *Scope Transform:* $\mathbf{A}^{/\sigma/} \;=\; \{x^t: \; (\exists s)\big(\, x \in_s \mathbf{A} \;\; \& \;\; s \in_t \sigma \,\big\}$.

EXAMPLES: $\{a^A, b^B, c^C\}^{/\{A^X, B^Y, C^Z\}/} = \{a^X, b^Y, c^Z\}, \;\; \text{and} \;\; z^{/\emptyset/} = \emptyset.$

Definition 1.6 *Element Projection:*

$$\boldsymbol{\rho}_s(\mathbf{A}) = x \iff (x \in_s \mathbf{A} \;\&\; (\forall a)(a \in_s \mathbf{A} \;\longrightarrow\; a = x)) \;\; or \;\; x = \perp, \;\; (\perp = undefined).$$

EXAMPLE: $\boldsymbol{\rho}_B(\{a^A, b^B, c^C\}) = b \;\;\&\;\; \boldsymbol{\rho}_B(\{a^B, b^B, c^C\}) = \perp$.

Definition 1.7 *Ordered Pair:* $< x, y > = \{x^1, y^2\}$.

Consequence 1.1 $\boldsymbol{\rho}_1(< x, y >) = x, \;\; \text{and} \;\; \boldsymbol{\rho}_2(< x, y >) = y.$

Definition 1.8 *n-Tuple:* $< x_1, x_2, ..., x_n > \;=\; \{\, x_1^1, x_2^2, ..., x_n^n \,\}$.

Note: $\boldsymbol{\rho}_i(< x_1, x_2, .., x_n >) \;=\; x_i, \;\; for \; 1 \le i \le n$

Definition 1.9 *Tuple of Natural Numbers (1 to n):* $\mathsf{N}(n) \;=\; \big\{\, x^x : \; n, x \in \mathbb{N} \;\&\; 1 \le x \le n \big\}$.

Definition 1.10 *Tuple of Natural Numbers (i+1 to i+n):* $\mathsf{N}(n, i) \;=\; \big\{\, (x+i)^x : \; i, n, x \in \mathbb{N} \;\&\; 1 \le x \le n \big\}$.

Note: $\mathsf{N}(n, 0) = \mathsf{N}(n) \;\; \text{and} \;\; \mathsf{N}(4, 6) \;= < 10, 11, 12, 13 >$.

Definition 1.11 *Subsets:*

$$\mathbf{A} \subset \mathbf{B} \iff \mathbf{A} \subseteq \mathbf{B} \;\&\; \mathbf{A} \ne \mathbf{B}, \;\; \text{and} \;\; \mathbf{A} \subseteq \mathbf{B} \iff (\forall x, s)\Big(\, x \in_s \mathbf{A} \;\to\; x \in_s \mathbf{B}\Big),$$

$$\mathbf{A} \Subset \mathbf{B} \iff \emptyset \ne \mathbf{A} \subset \mathbf{B}, \;\; \text{and} \;\; \mathbf{A} \subseteqq \mathbf{B} \iff \mathbf{A} \subseteq \mathbf{B} \;\;\&\;\; \mathbf{B} \ne \emptyset \to \mathbf{A} \ne \emptyset.$$

Definition 1.12 *Union:* $\mathbf{A} \cup \mathbf{B} \;=\; \{\, x^y\!: \; x \in_y \mathbf{A} \;\; or \;\; x \in_y \mathbf{B} \,\}$.

Definition 1.13 *Intersection:* $\mathbf{A} \cap \mathbf{B} \;=\; \{\, x^y\!: \; x \in_y \mathbf{A} \;\; and \;\; x \in_y \mathbf{B} \,\}$.

Definition 1.14 *Relative Complement:* $\mathbf{A} \;\sim\; \mathbf{B} = \{\, x^y\!: \; x \in_y \mathbf{A} \;\&\; x \notin_y \mathbf{B}\}$.

Definition 1.15 *Set Restriction:*

$$\mathbf{Q}\underset{\sigma}{|}\mathbf{A} \;=\; \Big\{\, z^w : (\exists\, a, s)\big(a \in_s \mathbf{A} \;\;\&\;\; z \in_w \mathbf{Q} \;\;\&\;\; s \cap w^{/\sigma/} = w^{/\sigma/} \ne \emptyset \;\;\&\;\; a \cap z^{/\sigma/} = z^{/\sigma/} \ne \emptyset)\Big\}.$$

Definition 1.16 *Domain Extraction:* $\boldsymbol{\eth}_\sigma(\mathbf{Q}) \;=\; \Big\{\, x^s\!: \;(\exists z, w)\big(\, z \in_w \mathbf{Q} \;\;\&\;\; x = z^{/\sigma/} \neq \emptyset \;\;\&\;\; s = w^{/\sigma/} \,\big) \Big\}.$

EXAMPLES: $\boldsymbol{\eth}_{\{A^1,C^2\}}\big(\{\{a^A, b^B, c^C\}\}\big) = \{\{a^1, c^2\}\},$

$\boldsymbol{\eth}_{<3,1>}\big(\{< a, b, c >^{<A,B,C>}\}\big) = \{< c, a >^{<C,A>}\},$

$\boldsymbol{\eth}_{\{3^1,1^2,y^9,v^5,v^7,Q^A\}}\big(\{< a, b, c >^{\{x^y, w^v, z^Q\}}\}\big) = \big\{< c, a >^{\{x^9, w^5, w^7, z^A\}}\big\}.$

Definition 1.17 *Image:* $\mathbf{Q}\big[\mathbf{A}\big]_\sigma = \;\; \boldsymbol{\eth}_{\sigma_2}(\mathbf{Q}\big|_{\sigma_1}\mathbf{A}), \quad (\sigma \;= < \sigma_1, \sigma_2 >).$

Tuples are sets having unique integer scopes from '1' to some 'n'. Formally, a set $\mathbf{x}$ is considered to be a 'tuple' iff there exists an n such that the scope set of $\mathbf{x}$ is equal to the set of natural numbers from 1 to n, [*i.e.* $\mathcal{S}(\mathbf{x}) = \mathbf{N}(n)$] and there are no duplicate scopes in $\mathbf{x}$. When the cardinality of the set $\mathbf{x}$ is equal to n, $\#(x) = n$, the set $\mathbf{x}$ is generally referred to as an 'n-tuple'.

Definition 1.18 *Tuple:* $tup(x) = n \iff \big(\#(x) = n \;\;\&\;\; \mathcal{S}(x) = \mathbf{N}(n)\big) \;\; or \;\; n = 0.$

This definition confines tuples to be finite which is in keeping with the traditional use and intent of tuples.

Definition 1.19 *Lazy Notation:* $tup(x_1, .., x_n) = k \implies (\forall i)(1 \le i \le n)(tup(x_i) = k).$
Definition 1.20 *Convention:* $tup(x) \iff tup(x) > 0.$
Definition 1.21 *Lazy Notation:* $tup(x_1, .., x_n) \implies (\forall i)(1 \le i \le n)(tup(x_i)).$
Definition 1.22 *Tuple Notation:* $< x_1, x_2, ..., x_n > \;=\; \{\, x_1^1, x_2^2, ..., x_n^n \,\}.$
Definition 1.23 *Convention: Given* $tup(x) = n$*, then for* $1 \le i \le n \;\; \boldsymbol{\rho}_i(x) = x_i.$

EXAMPLE: $tup(\sigma) = 2 \implies \sigma_1 = \boldsymbol{\rho}_1(\sigma) \;\;\&\;\; \sigma_2 = \boldsymbol{\rho}_2(\sigma).$

Definition 1.24 *Concatenation:* $x \cdot y \;=$

$$\Big\{ z^i\!: (\exists n, m)\Big(tup(x) = n \;\&\; tup(y) = m \;\&\; \big(1 \le i \le n \to z = \boldsymbol{\rho}_i(x)\big) \;\&\; \big(1 \le i - n \le m \to z = \boldsymbol{\rho}_{i-n}(y)\big)\Big)\Big\}.$$

EXAMPLE: $< x_1, .., x_n > \cdot < y_1, .., y_m > \;=\; < x_1, .., x_n, y_1, .., y_m > .$

Note: $tup(x) = n \;\;\&\;\; tup(y) = m \longrightarrow tup(x \cdot y) = n + m.$

Definition 1.25 $\emptyset \cdot x \;=\; x \cdot \emptyset \;=\; x.$
Assertion 1.1 $x \cdot y \cdot z \;=\; x \cdot (y \cdot z) \;=\; (x \cdot y) \cdot z.$
Definition 1.26 *Cartesian Product:* $\mathbf{A} \times \mathbf{B} \;=\; \big\{\, (x \cdot y)^{(s \cdot t)}\!: \; (x \in_s \mathbf{A} \;\;\&\;\; y \in_t \mathbf{B} \;\;\&\;\; tup(x, y, s, t)\big\}.$

2 TUPLESETS

Tuplesets are sets of tuples with tuples as scopes such that both are of the same arity.

Definition 2.1 *Tupleset:* $Tupset(\mathbf{A}) \iff (\forall x, s)\big(x \in_s \mathbf{A} \to (\exists n)(tup(x, s) = n)\big).$
Examples 2.1 $\{ < a, b >^{<6,9>}, \; < c >^{<A>} \}, \; \{ < a, b, < c, d >>^{<A,B,C>}\}, \; \{ < a, < b, < c >>>^{<\{w,v\},<1,2>>}\}.$

Consequences 2.1 *From Definitions:*

$(a)\; Tupset(\mathbf{A}) \;\;\&\;\; \mathbf{B} \Subset \mathbf{A} \;\;\to\;\; Tupset(\mathbf{B}),$

$(b)\; Tupset(\mathbf{A}) \;\;\&\;\; Tupset(\mathbf{B}) \;\;\to\;\; Tupset(\mathbf{A} \cup \mathbf{B}).$

Definition 2.2 *Lazy Notation:* $Tupset(\, \mathbf{A}_1, .., \mathbf{A}_n \,) \iff (\forall i)(1 \le i \le n)\,(Tupset(\, \mathbf{A}_i \,))\,.$
Definition 2.3 *Tupleset Degree:* $Tupset(\mathbf{A}) = k \iff (\forall x, s)\big(x \in_s \mathbf{A} \to tup(x, s) = k\big).$
Definition 2.4 *Lazy Notation:* $Tupset(\, \mathbf{A}_1, .., \mathbf{A}_n \,) = k \iff (\forall i)(1 \le i \le n)\,(Tupset(\, \mathbf{A}_i \,) = k)\,.$
Consequence 2.1 $Tupset(\mathbf{Q}) = n \;\;\&\;\; \mathbf{A} \Subset \mathbf{Q} \;\to\; Tupset(\mathbf{A}) = n.$

2.1 Tagged Sets:

The notation $\mathbf{f}_{(\sigma)}$ asserts that $\mathbf{f}$ is a tupleset, σ is a 2-tuple, and the image of $\mathbf{f}$ under σ is well defined. The set $\mathbf{f}$ is then said to be *tagged* by σ.

Definition 2.5 *Tagged Set:* $\mathbf{f}_{(\sigma)} \iff (\exists n)(Tupset(\mathbf{f}) = n) \;\;\&\;\; tup(\sigma) = 2 \;\;\&\;\; (\forall \mathbf{g})(\mathbf{g} \Subset \mathbf{f})(\exists x)\big(\, \mathbf{g}\big[x\big]_\sigma \neq \emptyset \,\big).$

The notation $\mathbf{f}_{(\sigma)}$ does not represent a set. The notation $\mathbf{f}_{(\sigma)}$ represents the *behavior* of the set $\mathbf{f}$ when *tagged* by the set σ. Nothing about the definition of a tagged set precludes it from having bizarre behavior. Refinements on the type of behavior are necessary in order to restrict the behavior to conform to some accepted notion of a *function*. The first step is to establish conditions under which two tagged sets exhibit the same behavior.

Definition 2.6 *Behavioral Equivalence:* $\mathbf{f}_{(\sigma)} = \mathbf{g}_{(\gamma)} \iff (\forall x)\big(\, \mathbf{f}\big[x\big]_\sigma = \mathbf{g}\big[x\big]_\gamma \,\big).$

The above equivalence does not assert a unique mathematical identity for set membership, but does assert a unique mathematical identity for set behavior.

Consequence 2.2 $\mathbf{f}_{(\sigma)} = \mathbf{g}_{(\gamma)} \longrightarrow Ɖ_{\sigma_1}(\mathbf{f}) = Ɖ_{\gamma_1}(\mathbf{g}) \;\;\&\;\; Ɖ_{\sigma_2}(\mathbf{f}) = Ɖ_{\gamma_2}(\mathbf{g}).$

Consequence 2.3 $\mathbf{f}_{(\sigma)} = \mathbf{g}_{(\gamma)} \;\;\&\;\; \mathbf{g}_{(\gamma)} = \mathbf{h}_{(\tau)} \longrightarrow \mathbf{f}_{(\sigma)} = \mathbf{h}_{(\tau)}.$

Consequence 2.4 *Negative Properties:*

(a) $\mathbf{f}_{(\sigma)} = \mathbf{g}_{(\sigma)} \;\not\Rightarrow\; \mathbf{f} = \mathbf{g}.$

Ex: $\mathbf{f} = \{< a, b, x >\},\ \mathbf{g} = \{< a, b, w >\},\ \sigma =<< 1 >, < 2 >>$.

(b) $\mathbf{f}_{(\sigma)} = \mathbf{f}_{(\gamma)} \;\not\Rightarrow\; \sigma = \gamma.$

Ex: $\mathbf{f} = \{< a, b >, < b, a >\},\ \sigma =<< 1 >, < 2 >>,\ \gamma =<< 2 >, < 1 >>$.

(c) $\mathbf{f}_{(\sigma)} = \mathbf{f}_{(\gamma)} \;\&\; \mathbf{g} \Subset \mathbf{f} \;\not\Rightarrow\; \mathbf{g}_{(\sigma)} = \mathbf{g}_{(\gamma)}.$

Ex: $\mathbf{f} = \{< a, b >, < b, a >\},\ \mathbf{g} = \{< a, b >\},\ \sigma =<< 1 >, < 2 >>,\ \gamma =<< 2 >, < 1 >>$.

(d) $\mathbf{f}_{(\sigma)} = \mathbf{g}_{(\gamma)} \;\&\; \mathbf{f} \in_\sigma \mathbf{A} \;\not\Rightarrow\; \mathbf{g} \in_\gamma \mathbf{A}$.

Ex: $\mathbf{A} = \{< a, b >^{<<1>,<2>>}\},\ \mathbf{B} = \{< b, a >^{<<2>,<1>>}\},\ \mathbf{f} \in_\sigma \mathbf{A},\ \&\ \mathbf{g} \in_\gamma \mathbf{B}.$

Definition 2.7 *Null Behavior:* $\mathbf{f}_{(\sigma)} = \emptyset_{(\emptyset)} \iff (\forall x)\big(\, \mathbf{f}\big[x\big]_\sigma = \emptyset \,\big).$

This paper distinguishes how a set behaves from the result set produced by its behavior. However, all definitions are required to be in terms of set membership. The following definition of *application* (which unites the action defined by the behavior of a tagged set with the result set produced) is just a restricted use of the already defined image operation (Def. 1.17) to tuplesets.

Definition 2.8 *Application:* $\mathbf{f}_{(\sigma)}(x) = \mathbf{f}\big[x\big]_\sigma.$

An application dictates a *process* that produces a result set, when the process is applied to another set. It is important to notice that the application of a tagged set produces a set while a tagged set itself defines a behavior of the set being tagged, and does not define a specific set.

Observation: '$\mathbf{f}_{(\sigma)}(x) \in_s \mathbf{Q}$' makes mathematical sense, while '$\mathbf{f}_{(\sigma)} \in_s \mathbf{Q}$' does not make mathematical sense. The expression '$\mathbf{f}_{(\sigma)}(x)$' defines a set-membership condition, while the expression '$\mathbf{f}_{(\sigma)}$' defines a set-behavior.

Consequence 2.5 *Application Properties:*

$$(a)\ \big(\mathbf{f} \cup \mathbf{g}\big)_{(\sigma)}(x) = \mathbf{f}_{(\sigma)}(x) \cup \mathbf{g}_{(\sigma)}(x),$$
$$(b)\ \big(\mathbf{f} \cap \mathbf{g}\big)_{(\sigma)}(x) \subseteq \mathbf{f}_{(\sigma)}(x) \cap \mathbf{g}_{(\sigma)}(x),$$
$$(c)\ \mathbf{f}_{(\sigma)}(x) \sim \mathbf{g}_{(\sigma)}(x) \subseteq \big(\mathbf{f} \sim \mathbf{g}\big)_{(\sigma)}(x).$$

Examples 2.2

Given: $\mathbf{f} = \Big\{ < a, x >^{<A,Z>},\ < b, y >^{<B,Y>},\ < c, x >^{<A,Z>} \Big\},$

(a) $\mathbf{f}_{(\sigma)}\big(\{<a>^{<A>}\}\big) = \mathbf{f}\big[\{<a>^{<A>}\}\big]_\sigma = \{<x>^{<Z>}\},\ \textit{with}\ \sigma =<< 1 >, < 2 >>,$

(b) $\mathbf{f}_{(\tau)}\big(\{<x>^{<Z>}\}\big) = \mathbf{f}\big[\{<x>^{<Z>}\}\big]_\tau = \{<a>^{<A>},\ <c>^{<A>}\},\ \textit{with}\ \tau =<< 2 >, < 1 >>,$

(c) $\mathbf{f}_{(\alpha)}(\mathbf{f}) = \mathbf{f}\big[\mathbf{f}\big]_\alpha = \{<x, a>^{<Z,A>},\ <x, c>^{<Z,A>},\ <y, b>^{<Y,B>}\},\ \textit{with}\ \alpha =<< 1, 2 >, < 2, 1 >>,$

(d) $\big(\mathbf{f} \times \mathbf{f}_{(\alpha)}(\mathbf{f})\big)_{(\mu)}(\mathbf{f}) = \big(\mathbf{f} \times \mathbf{f}\big[\mathbf{f}\big]_\alpha\big)\big[\mathbf{f}\big]_\mu =$

$$\{<a, a>^{<A,A>},\ <a, c>^{<A,A>},\ <b, b>^{<B,B>},\ <c, a>^{<A,A>},\ <c, c>^{<A,A>}\},$$

$$\textit{with}\ \alpha =<< 1, 2 >, < 2, 1 >> \;\;\&\;\; \mu =< \{2^2, 3^2\}, < 1, 4 >>,$$

The last two examples above are instances of *self-application*, about which more will be said subsequently.

Definition 2.9 *Tagged Application:* $\mathbf{f}_{(\sigma)}(\mathbf{g})_{(\tau)} \; = \; \Big(\mathbf{f}_{(\sigma)}(\mathbf{g})\Big)_{(\tau)}.$

Notice that $\mathbf{g}_{(\tau)}$ may or may not make sense, but whether or not it does is not relevant to the definition. Note also a *tagged application* produces a tagged set, not a result set!

Consequence 2.6 *Natural Tagged Application Combinations: 0-Intermediates*

$$(a)\;\; \mathbf{f}_{(\sigma)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\mathbf{f}_{(\sigma)}(\mathbf{g})\Big)_{(\tau)}(x) \; = \; \Big(\mathbf{f}\big[\mathbf{g}\big]_{\sigma}\Big)\big[x\big]_{\tau}.$$

Consequence 2.7 *Natural Tagged Application Combinations: 1-Intermediate*

$$(a)\;\; \mathbf{f}_{(\sigma)}\mathbf{a}_{(s)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\mathbf{f}_{(\sigma)}\big(\mathbf{a}_{(s)}(\mathbf{g})\big)\Big)_{(\tau)}(x) \; = \; \Big(\mathbf{f}\big[\mathbf{a}\big[\mathbf{g}\big]_{s}\big]_{\sigma}\Big)\big[x\big]_{\tau},$$

$$(b)\;\; \mathbf{f}_{(\sigma)}(\mathbf{a})_{(s)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}_{(\sigma)}(\mathbf{a})\big)_{(s)}(\mathbf{g})\Big)_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}\big[\mathbf{a}\big]_{\sigma}\big)\big[\mathbf{g}\big]_{s}\Big)\big[x\big]_{\tau}.$$

Consequence 2.8 *Natural Tagged Application Combinations: 2-Intermediates*

$$(a)\;\; \mathbf{f}_{(\sigma)}\mathbf{a}_{(s)}\mathbf{b}_{(t)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\mathbf{f}_{(\sigma)}\big(\mathbf{a}_{(s)}\big(\mathbf{b}_{(t)}(\mathbf{g})\big)\big)\Big)_{(\tau)}(x) \; = \; \Big(\mathbf{f}\big[\mathbf{a}\big[\mathbf{b}\big[\mathbf{g}\big]_{t}\big]_{s}\big]_{\sigma}\Big)\big[x\big]_{\tau},$$

$$(b)\;\; \mathbf{f}_{(\sigma)}\mathbf{a}_{(s)}(\mathbf{b})_{(t)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}_{(\sigma)}\big(\mathbf{a}_{(s)}(\mathbf{b})\big)\big)_{(t)}(\mathbf{g})\Big)_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}\big[\mathbf{a}\big[\mathbf{b}\big]_{s}\big]_{\sigma}\big)\big[\mathbf{g}\big]_{t}\Big)\big[x\big]_{\tau},$$

$$(c)\;\; \mathbf{f}_{(\sigma)}(\mathbf{a})_{(s)}\mathbf{b}_{(t)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}_{(\sigma)}(\mathbf{a})\big)_{(s)}\big(\mathbf{b}_{(t)}(\mathbf{g})\big)\Big)_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}\big[\mathbf{a}\big]_{\sigma}\big)\big[\mathbf{b}\big[\mathbf{g}\big]_{t}\big]_{s}\Big)\big[x\big]_{\tau},$$

$$(d)\;\; \mathbf{f}_{(\sigma)}(\mathbf{a})_{(s)}(\mathbf{b})_{(t)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\big(\mathbf{f}_{(\sigma)}(\mathbf{a})\big)_{(s)}(\mathbf{b})\big)_{(t)}(\mathbf{g})\Big)_{(\tau)}(x) \; = \; \Big(\big(\big(\mathbf{f}\big[\mathbf{a}\big]_{\sigma}\big)\big[\mathbf{b}\big]_{s}\big)\big[\mathbf{g}\big]_{t}\Big)\big[x\big]_{\tau}.$$

Consequence 2.9 *Natural Tagged Application Combinations: 3-Intermediates*

$$(a)\;\; \mathbf{f}_{(\sigma)}\mathbf{a}_{(s)}\mathbf{b}_{(t)}\mathbf{c}_{(v)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\mathbf{f}_{(\sigma)}\big(\mathbf{a}_{(s)}\big(\mathbf{b}_{(t)}\big(\mathbf{c}_{(v)}(\mathbf{g})\big)\big)\big)\Big)_{(\tau)}(x) \; = \; \Big(\mathbf{f}\big[\mathbf{a}\big[\mathbf{b}\big[\mathbf{c}\big[\mathbf{g}\big]_{v}\big]_{t}\big]_{s}\big]_{\sigma}\Big)\big[x\big]_{\tau},$$

$$(b)\;\; \mathbf{f}_{(\sigma)}\mathbf{a}_{(s)}\mathbf{b}_{(t)}(\mathbf{c})_{(v)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}_{(\sigma)}\big(\mathbf{a}_{(s)}\big(\mathbf{b}_{(t)}(\mathbf{c})\big)\big)\big)_{(v)}(\mathbf{g})\Big)_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}\big[\mathbf{a}\big[\mathbf{b}\big[\mathbf{c}\big]_{t}\big]_{s}\big]_{\sigma}\big)\big[\mathbf{g}\big]_{v}\Big)\big[x\big]_{\tau},$$

$$(c)\;\; \mathbf{f}_{(\sigma)}\mathbf{a}_{(s)}(\mathbf{b})_{(t)}\mathbf{c}_{(v)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}_{(\sigma)}\big(\mathbf{a}_{(s)}(\mathbf{b})\big)\big)_{(t)}\big(\mathbf{c}_{(v)}(\mathbf{g})\big)\Big)_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}\big[\mathbf{a}\big[\mathbf{b}\big]_{s}\big]_{\sigma}\big)\big[\mathbf{c}\big[\mathbf{g}\big]_{v}\big]_{t}\Big)\big[x\big]_{\tau},$$

$$(d)\;\; \mathbf{f}_{(\sigma)}\mathbf{a}_{(s)}(\mathbf{b})_{(t)}(\mathbf{c})_{(v)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\big(\mathbf{f}_{(\sigma)}\big(\mathbf{a}_{(s)}(\mathbf{b})\big)\big)_{(t)}(\mathbf{c})\big)_{(v)}(\mathbf{g})\Big)_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}\big[\mathbf{a}\big[\mathbf{b}\big]_{s}\big]_{\sigma}\big)\big[\mathbf{c}\big]_{t}\big)\big[\mathbf{g}\big]_{v}\Big)\big[x\big]_{\tau},$$

$$(e)\;\; \mathbf{f}_{(\sigma)}(\mathbf{a})_{(s)}\mathbf{b}_{(t)}\mathbf{c}_{(v)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}_{(\sigma)}(\mathbf{a})\big)_{(s)}\big(\mathbf{b}_{(t)}\big(\mathbf{c}_{(v)}(\mathbf{g})\big)\big)\Big)_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}\big[\mathbf{a}\big]_{\sigma}\big)\big[\mathbf{b}\big[\mathbf{c}\big[\mathbf{g}\big]_{v}\big]_{t}\big]_{s}\Big)\big[x\big]_{\tau},$$

$$(f)\;\; \mathbf{f}_{(\sigma)}(\mathbf{a})_{(s)}\mathbf{b}_{(t)}(\mathbf{c})_{(v)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\big(\mathbf{f}_{(\sigma)}(\mathbf{a})\big)_{(s)}\big(\mathbf{b}_{(t)}(\mathbf{c})\big)\big)_{(v)}(\mathbf{g})\Big)_{(\tau)}(x) \; = \; \Big(\big(\big(\mathbf{f}\big[\mathbf{a}\big]_{\sigma}\big)\big[\mathbf{b}\big[\mathbf{c}\big]_{t}\big]_{s}\big)\big[\mathbf{g}\big]_{v}\Big)\big[x\big]_{\tau},$$

$$(g)\;\; \mathbf{f}_{(\sigma)}(\mathbf{a})_{(s)}(\mathbf{b})_{(t)}\mathbf{c}_{(v)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\big(\mathbf{f}_{(\sigma)}(\mathbf{a})\big)_{(s)}(\mathbf{b})\big)_{(t)}\big(\mathbf{c}_{(v)}(\mathbf{g})\big)\Big)_{(\tau)}(x) \; = \; \Big(\big(\mathbf{f}\big[\mathbf{a}\big]_{\sigma}\big)\big[\mathbf{b}\big]_{s}\big)\big[\mathbf{c}\big[\mathbf{g}\big]_{v}\big]_{t}\Big)\big[x\big]_{\tau},$$

$$(h)\;\; \mathbf{f}_{(\sigma)}(\mathbf{a})_{(s)}(\mathbf{b})_{(t)}(\mathbf{c})_{(v)}(\mathbf{g})_{(\tau)}(x) \; = \; \Big(\big(\big(\big(\mathbf{f}_{(\sigma)}(\mathbf{a})\big)_{(s)}(\mathbf{b})\big)_{(t)}(\mathbf{c})\big)_{(v)}(\mathbf{g})\Big)_{(\tau)}(x) \; = \; \Big(\big(\big(\big(\mathbf{f}\big[\mathbf{a}\big]_{\sigma}\big)\big[\mathbf{b}\big]_{s}\big)\big[\mathbf{c}\big]_{t}\big)\big[\mathbf{g}\big]_{v}\Big)\big[x\big]_{\tau}.$$

There are no caveats that preclude any of the above from using the same tupleset in all variable locations. It would not be surprising if such a substitution always yielded a null result, though it might be surprising that such a substitution generally does not yield a null result. Consider the following example for substitution in 2.8(d) above.

Example 2.1 *Sever Self-Application:*

$$Let\;\; \mathbf{f} \; = \; \{ < a,b,c,d >^{<A,B,C,D>}, \; < w,x,y,z >^{<W,X,Y,Z>} \}, \;\; with$$
$$\gamma = << 1 >, < 2,3,4,1 >>, \quad \sigma = << 4 >, < 2,3,4,1 >>, \quad \tau = << 3 >, < 2,1 >>, \;\; then$$
$$\mathbf{f}_{(\gamma)}(\mathbf{f})_{(\sigma)}(\mathbf{f})_{(\tau)}(\mathbf{f}) \; = \; \{ < d,c >^{<D,C>}, \; < z,y >^{<Z,Y>} \}.$$

Collections of sets of like behavior can be achieved by restricting the types of tagged sets, by specifying the domain of sets allowable for applications, and by delineating the range of allowable sets produced. These collections will be defined as *process spaces*.

3 SUPPLEMENT: (assuming a working definition for application)

Definition 3.1 *Total Domain:* $\mathbf{Đ}_*(\mathbf{Q}) \ = \ \Big\{ \ x^s: \ (\exists k)(Tupset(\ \mathbf{Q}) = k) \ \ \& \ \ (1 \leq i \leq k) \ \ \& \ \ x \in_s \mathbf{Đ}_{<i>}(\mathbf{Q}) \ ) \Big\}.$

EXAMPLE: $\mathbf{Đ}_*\big(\{<a,b>^{<A,B>}, <x,y>^{<X,Y>}\big) = \{<a>^{<A>}, <b>^{<B>}, <x>^{<X>}, <y>^{<Y>}\}.$

3.1 Process Space:

A tagged set, $\mathbf{P}$, behaves as a *process* from $\mathbf{A}$ to $\mathbf{B}$ *under* σ if and only if $<\mathbf{P}> \in_{<\sigma>} Prs(\mathbf{A},\mathbf{B})$, as defined below.

Definition 3.2 *Prs-Space:*

$$Prs(\mathbf{A},\mathbf{B}) \ = \ \Big\{ <\mathbf{P}>^{<\sigma>}: \ \big(\ Tupset(\mathbf{A},\mathbf{B},\mathbf{P}) \ \& \ tup(\sigma) = 2 \ \& \ \sigma_1 = \boldsymbol{\rho}_1(\sigma) \ \& \ \sigma_2 = \boldsymbol{\rho}_2(\sigma) \ \big) \ \& \ \big(\mathbf{Đ}_*(\mathbf{P}) \ \Subset \ \mathbf{Đ}_*(\mathbf{A} \cup \mathbf{B}) \ \& \ \mathbf{Đ}_{\sigma_1}(\mathbf{P}) \ \Subset \ \mathbf{A} \ \& \ \mathbf{Đ}_{\sigma_2}(\mathbf{P}) \ \Subset \ \mathbf{B} \ \big) \ \& \ (\forall x)\big(\ \mathbf{P}_{(\sigma)}(x) \ \subseteq \ \mathbf{B} \ \big) \Big\}.$$

A *process* in XST is defined in terms of *inputs* and *outputs*, where the inputs are sets of n-tuples and the outputs are sets of m-tuples. Though not essential, the above definition was chosen to preclude *source* and *terminal* processes. That is that neither the input domain nor output domain can be null. This definition also asserts that every process on $\mathbf{A}$ produces a non-empty result and that all results are confined to $\mathbf{B}$.

3.2 Process Composition:

Given that tagged sets define a process, under what conditions can tagged sets be combined to define a composite process? In most set theories, this is done (for functions) through the definition of *composition*.

Definition 3.3 *Composition:* $\mathbf{h}_{(\tau)} \ = \ \mathbf{g}_{(\gamma)} \circ \mathbf{f}_{(\sigma)} \iff (\forall x)\big(\ \mathbf{h}\big[x\big]_\tau \ = \ \mathbf{g}\big[\mathbf{f}\big[x\big]_\sigma\big]_\gamma \big).$

Assertion 3.1 *Associativity:* $\mathbf{h}_{(\tau)} \circ \mathbf{g}_{(\gamma)} \circ \mathbf{f}_{(\sigma)} \ = \ \mathbf{h}_{(\tau)} \circ (\mathbf{g}_{(\gamma)} \circ \mathbf{f}_{(\sigma)}) \ = \ (\mathbf{h}_{(\tau)} \circ \mathbf{g}_{(\gamma)}) \circ \mathbf{f}_{(\sigma)}.$

Assertion 3.2 *Substitution:* $\mathbf{h}_{(\tau)} = \mathbf{g}_{(\gamma)} \circ \mathbf{f}_{(\sigma)}, \ \mathbf{g}_{(\gamma)} = \mathbf{j}_{(\eta)} \ \& \ \mathbf{f}_{(\sigma)} = \mathbf{k}_{(\nu)} \longrightarrow \mathbf{h}_{(\tau)} = \mathbf{j}_{(\eta)} \circ \mathbf{k}_{(\nu)}.$

Consequence 3.1 $(\mathbf{g}_{(\gamma)} \circ \mathbf{f}_{(\sigma)})(x) \ = \ \mathbf{g}\big[\mathbf{f}\big[x\big]_\sigma\big]_\gamma \ = \ \mathbf{g}_{(\gamma)}\Big(\mathbf{f}_{(\sigma)}(x)\Big).$

Consequence 3.2 $(\mathbf{g}_{(\gamma)} \circ \mathbf{f}_{(\sigma)})\,|_\gamma\mathbf{A} \ = \ \mathbf{g}_{(\gamma)} \circ \big(\mathbf{f}\,|_\gamma\mathbf{A}\big)_{(\sigma)}.$

Though these definitions may make mathematical sense when the compositions exists, they give no assurance that a specific composition will actually exist.

Assertion 3.3 $(\exists x)\Big(\mathbf{g}_{(\gamma)}\big(\mathbf{f}_{(\sigma)}(x)\ \big)\Big) \neq \emptyset \implies \mathbf{Đ}_{\sigma_2}(\mathbf{f}) \ \Subset \ \mathbf{Đ}_{\gamma_1}(\mathbf{g}).$

3.3 Identity Process:

All Tuplesets have a unique property in that they can define their own identity process.

Definition 3.4 $Tupset(\mathbf{A}) = k \implies @\mathbf{A} = <\mathsf{N}(k), \mathsf{N}(k)> .$

Definition 3.5 $Tupset(\mathbf{A}) \implies \mathbf{I}_\mathbf{A} \ = \ \mathbf{A}_{(@\mathbf{A})}.$

Consequences 3.1 $For \ \ Tupset(\mathbf{A}):$

$(a) \ \ x \Subset \mathbf{A} \implies \mathbf{I}_\mathbf{A}(x) = x,$

$(b) \ \ <\mathbf{A}> \ \in_{<@\mathbf{A}>} Prs(\mathbf{A},\mathbf{A}),$

$(c) \ \ \mathbf{I}_\mathbf{A}(\mathbf{A}) = \mathbf{A}.$

Assertion 3.4 *For all* $<\mathbf{P}> \in_{<\sigma>} Prs(\mathbf{Q},\mathbf{Q}), \ \ \mathbf{P}_{(\sigma)} \ = \ \mathbf{P}_{(\sigma)} \circ \mathbf{I}_\mathbf{Q} \ = \ \mathbf{I}_\mathbf{Q} \circ \mathbf{P}_{(\sigma)}.$

3.4 Normalized Process:

For any given $\mathbf{Q}$ a process space can be arbitrarily large, since any specific behavior can have an infinite number of representations. The size of any given process space can be reduced to a minimum by excluding all but a unique canonical representation for each unique behavior, if of course such a unique representation exists. The next definition of a *normalized process* will provide such an existence.

Definition 3.6 *Normalized Tagged Set:* $(\mathbf{f}_{(\sigma)})^* \ = \ \mathbf{g}_{(!\sigma!)} \ \& \ Tupset(\mathbf{g}) = tup(\sigma_1) + tup(\sigma_2), \ \ with$
$!\sigma! \ = <\mathsf{N}(tup(\sigma_1)), \mathsf{N}(tup(\sigma_2), tup(\sigma_1))> .$

Consequence 3.3 $(\mathbf{f}_{(\sigma)})^* = \mathbf{g}_{(!\sigma!)} \implies \mathbf{g} = \{z^w: (\exists x,s)(\ x \in_s \mathbf{f} \ \& \ z = x^{/\sigma_1/} \cdot x^{/\sigma_2/} \ \& \ w = s^{/\sigma_1/} \cdot s^{/\sigma_2/} \ \}.$

Consequence 3.4 $\mathbf{g}_{(\tau)} = \mathbf{f}_{(\sigma)} \iff (\ \mathbf{g}_{(\tau)})^* \ = \ (\ \mathbf{f}_{(\sigma)})^*.$

4 XST FUNCTIONS

A tagged set, $\mathbf{f}$, behaves as a *function* from $\mathbf{A}$ to $\mathbf{B}$ *under* σ, written as $\mathbf{f}_{(\sigma)}: \mathbf{A} \to \mathbf{B}$, iff $<\mathbf{f}> \in_{<\sigma>} \mathcal{F}(\mathbf{A},\mathbf{B})$.

Definition 4.1 *$\mathcal{F}$-Space:*

$$\mathcal{F}(\mathbf{A},\mathbf{B}) \;=\; \Big\{ <\mathbf{f}>^{<\sigma>} : <\mathbf{f}> \in_{<\sigma>} Prs(\mathbf{A},\mathbf{B}) \;\&\; (\forall y)\Big(\; Sing(y) \;\&\; \mathbf{f}_{(\sigma)}(y) \neq \emptyset \;\rightarrow\; Sing(\mathbf{f}_{(\sigma)}(y)) \;\Big) \Big\}.$$

Three specific properties of functions combine to provide eight unique subsets of any given $\mathcal{F}$-Space, some of which are traditionally more interesting than others. They are: *on*, *onto*, and *one-to-one*.

Definition 4.2 *$\mathcal{F}$-Space ON $\mathbf{A}$ from $\mathbf{A}$ to $\mathbf{B}$:* $\mathcal{F}[\mathbf{A},\mathbf{B}) = \{<\mathbf{f}>^{<\sigma>} : <\mathbf{f}> \in_{<\sigma>} \mathcal{F}(\mathbf{A},\mathbf{B}) \;\&\; \text{Đ}_{\sigma_1}(\mathbf{f}) = \mathbf{A}\}$.

Definition 4.3 *$\mathcal{F}$-Space ONTO $\mathbf{B}$ from $\mathbf{A}$ to $\mathbf{B}$:* $\mathcal{F}(\mathbf{A},\mathbf{B}] = \{<\mathbf{f}>^{<\sigma>} : <\mathbf{f}> \in_{<\sigma>} \mathcal{F}(\mathbf{A},\mathbf{B}) \;\&\; \text{Đ}_{\sigma_2}(\mathbf{f}) = \mathbf{B}\}$.

Definition 4.4 *$\mathcal{F}$-Space 1-1 from $\mathbf{A}$ to $\mathbf{B}$:* $\mathcal{F}_1(\mathbf{A},\mathbf{B}) = \{<\mathbf{f}>^{<\sigma>} : <\mathbf{f}> \in_{<\sigma>} \mathcal{F}(\mathbf{A},\mathbf{B}) \;\&\; (\forall x, y)$
$(\; Sing(x,y) \;\&\; \mathbf{f}\big[x\big]_\sigma = \mathbf{f}\big[y\big]_\sigma \neq \emptyset \;\rightarrow\; x = y \;)\}$.

For any given $\mathbf{A}$ and $\mathbf{B}$ the set $\mathcal{F}(\mathbf{A},\mathbf{B})$, if non-empty, will be quite large, containing many different tagged sets, many of which exhibit exactly the same behavior. Through a *normalization* process, any $\mathcal{F}$-Space can be reduced to a canonical set of functions, preserving all the behaviors of the original $\mathcal{F}$-Space, but now with unique representatives.

Definition 4.5 *Normalized $\mathcal{F}$-Space:*

$$\mathcal{F}^*(\mathbf{A},\mathbf{B}) \;=\; \{ <\mathbf{f}>^{<\sigma>} : \; (\exists\, \mathbf{g}, \alpha)(<\mathbf{g}> \in_{<\alpha>} \mathcal{F}(\mathbf{A},\mathbf{B}) \quad \& \quad \mathbf{f}_{(\sigma)} = (\, \mathbf{g}_{(\alpha)})^* \;\&\; \sigma = !\alpha!)\}.$$

Consequences 4.1 *$\mathcal{F}^*$-Space Properties:*

(a) $\;<\mathbf{f}> \in_{<\sigma>} \mathcal{F}^*(\mathbf{A},\mathbf{B}) \;\;\&\;\; <\mathbf{g}> \in_{<\gamma>} \mathcal{F}^*(\mathbf{A},\mathbf{B}) \;\;\&\;\; \mathbf{f}_{(\sigma)} = \mathbf{g}_{(\gamma)} \;\;\Longrightarrow\;\; \mathbf{f} = \mathbf{g} \;\&\; \sigma = \gamma,$

(b) $\;\mathcal{F}^*[\mathbf{A},\mathbf{B}) \;\subseteq\; \mathcal{F}^*(\mathbf{A},\mathbf{B}),$

(c) $\;\mathcal{F}^*(\mathbf{A},\mathbf{B}] \;\subseteq\; \mathcal{F}^*(\mathbf{A},\mathbf{B}),$

(d) $\;\mathcal{F}^*[\mathbf{A},\mathbf{B}] \;\subseteq\; \mathcal{F}^*(\mathbf{A},\mathbf{B}).$

As can be seen from Figure 1, there are sixteen different classes of $\mathcal{F}$-Spaces. Eight of which are normalized, thus giving unique representation to specific behavior. Though all eight of these have CST equivalencies, only three are generally of traditional interest.

Definition 4.6 *Injective $\mathcal{F}$-Space:* $\mathcal{F}_1^*[\mathbf{A},\mathbf{B})$.
Definition 4.7 *Surjective $\mathcal{F}$-Space:* $\mathcal{F}^*(\mathbf{A},\mathbf{B}]$.
Definition 4.8 *Bijective $\mathcal{F}$-Space:* $\mathcal{F}_1^*[\mathbf{A},\mathbf{B}]$.

Consequences 4.2 *Subset Properties: For* $<\mathbf{f}> \in_{<\sigma>} \mathcal{F}(\mathbf{A},\mathbf{B})$ *and* $\mathbf{g} \,\dot{\subseteq}\, \mathbf{f}$,

(a) *if* $\mathbf{f}_{(\sigma)}$ *is* Injective *from* $\mathbf{A} \to \mathbf{B}$, *then* $\mathbf{g}_{(\sigma)}$ *is* Injective *from* $\text{Đ}_{\sigma_1}(\mathbf{g}) \to \mathbf{B}$,

(b) *if* $\mathbf{f}_{(\sigma)}$ *is* Surjective *from* $\mathbf{A} \to \mathbf{B}$, *then* $\mathbf{g}_{(\sigma)}$ *is* Surjective *from* $\mathbf{A} \to \text{Đ}_{\sigma_2}(\mathbf{g})$,

(c) *if* $\mathbf{f}_{(\sigma)}$ *is* Bijective *from* $\mathbf{A} \to \mathbf{B}$, *then* $\mathbf{g}_{(\sigma)}$ *is* Bijective *from* $\text{Đ}_{\sigma_1}(\mathbf{g}) \to \text{Đ}_{\sigma_2}(\mathbf{g})$.

The following properties are only presented for the most general cases, the top node in Figure 1. These will all hold if one adheres to any single node, but more complex results are generated when nodes are mixed.

Consequences 4.3 *$\mathcal{F}$-Space Boolean Properties:*

(a) $\;\mathcal{F}(\mathbf{A},\mathbf{B}) \;\subseteq\; Prs(\mathbf{A},\mathbf{B}),$

(b) $\;\mathcal{F}[\mathbf{A},\mathbf{B}) \;\subseteq\; \mathcal{F}(\mathbf{A},\mathbf{B}),$

(c) $\;\mathcal{F}(\mathbf{A},\mathbf{B}] \;\subseteq\; \mathcal{F}(\mathbf{A},\mathbf{B}),$

(d) $\;\mathcal{F}[\mathbf{A},\mathbf{B}] \;\subseteq\; \mathcal{F}(\mathbf{A},\mathbf{B}),$

(e) $\;\mathcal{F}(\mathbf{A},\mathbf{B}) \;\dot{\subseteq}\; \mathcal{F}(\mathbf{C},\mathbf{D}) \;\;\Longleftrightarrow\;\; \mathbf{A} \,\dot{\subseteq}\, \mathbf{C} \;\&\; \mathbf{B} \,\dot{\subseteq}\, \mathbf{D},$

(f) $\;\mathcal{F}(\mathbf{A},\mathbf{B}) \,\cup\, \mathcal{F}(\mathbf{C},\mathbf{D}) \;\subseteq\; \mathcal{F}(\mathbf{A}\cup\mathbf{C}, \mathbf{B}\cup\mathbf{D}),$

(g) $\;\mathcal{F}(\mathbf{A},\mathbf{B}) \,\cap\, \mathcal{F}(\mathbf{C},\mathbf{D}) \;=\; \mathcal{F}(\mathbf{A}\cap\mathbf{C}, \mathbf{B}\cap\mathbf{D}),$

(h) $\;\mathcal{F}(\mathbf{A}\sim\mathbf{C}, \mathbf{B}\sim\mathbf{D}) \;\subseteq\; \mathcal{F}(\mathbf{A},\mathbf{B}) \;\sim\; \mathcal{F}(\mathbf{C},\mathbf{D}).$

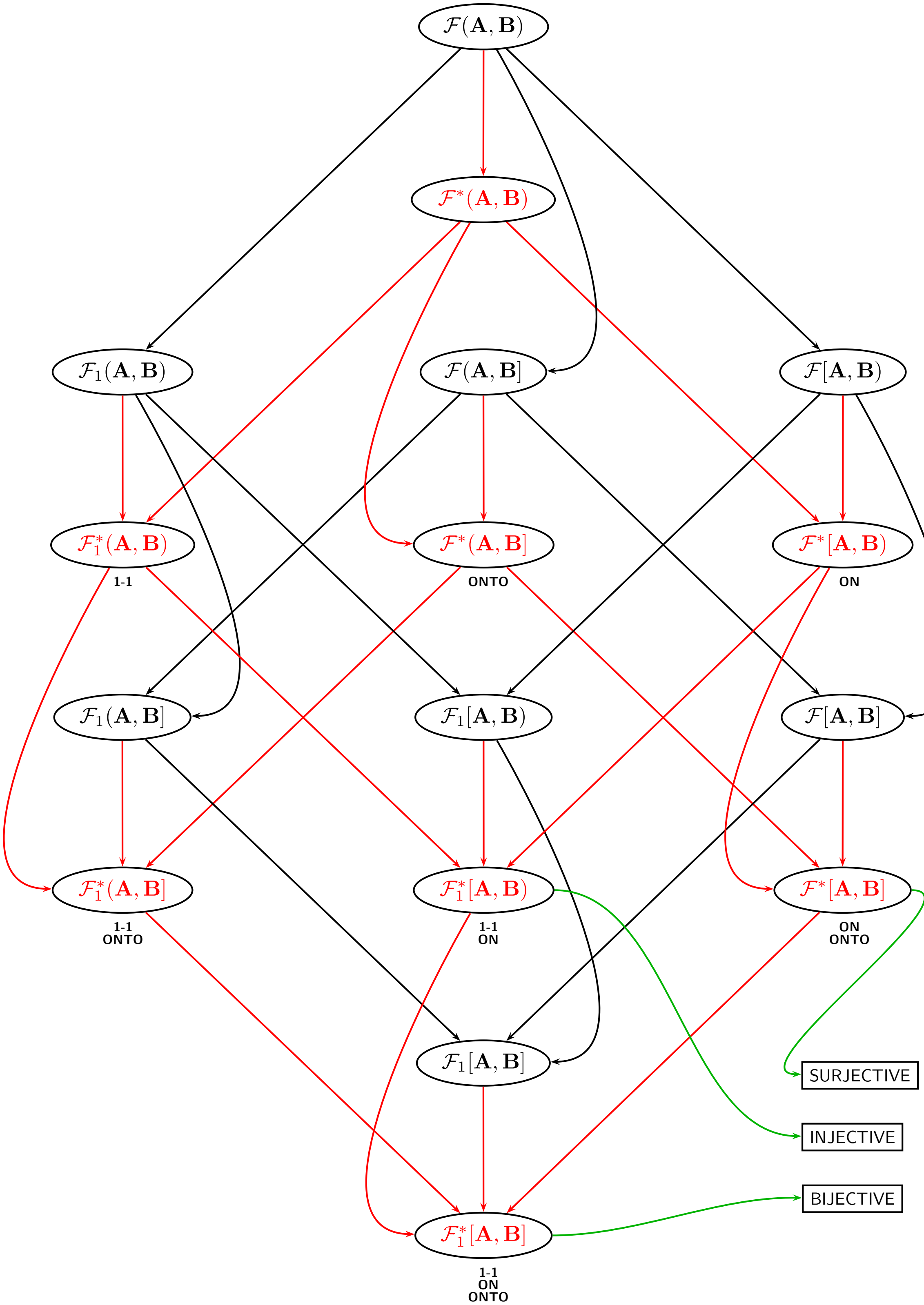
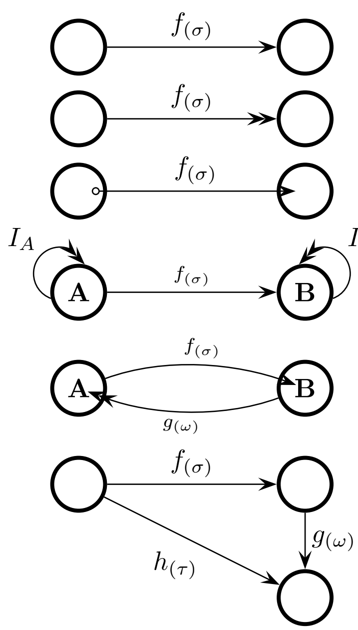


FIGURE 1 : $\mathcal{F}$-Spaces

5 DF-DIAGRAMS & KATEGORIES

Diagrams are often used in mathematics to better convey conceptual relationships burred beneath arcane notation. *DF-Diagrams*, or Domain-Function diagrams, are intended to pictorially convey certain interesting property preserving relationships regarding the behavior of functions. DF-Diagrams are intended to preserve all the interesting properties conveyed by the arrow diagrams of Category theory.

After establishing the connection between DF-Diagrams and their equivalent set-theoretic interpretations, a definition of *Kategory* as a specific type of tupleset will be given that defines a set of objects and functions between these objects. Kategories are intended to preserve the behavioral characteristics of Categories. Whether or not this objective will be successful, remains to be seen.



a) $\Longrightarrow \; <f> \in_{<\sigma>} \mathcal{F}^*[\,\text{Đ}_{\sigma_1}(f), \text{Đ}_{\sigma_2}(f)\,]$

b) $\Longrightarrow \; <f> \in_{<\sigma>} \mathcal{F}_1^*[\,\text{Đ}_{\sigma_1}(f), \text{Đ}_{\sigma_2}(f)\,]$

c) $\Longrightarrow \; <f> \in_{<\sigma>} \mathcal{F}^*(\,\mathbf{A}, \mathbf{B}\,), \quad \mathbf{A} \subseteq \text{Đ}_{\sigma_1}(f), \quad \mathbf{B} \subseteq \text{Đ}_{\sigma_2}(f)$

d) $\Longrightarrow \; <f> \in_{<\sigma>} \mathcal{F}^*[\,\mathbf{A}, \mathbf{B}\,], \quad \mathbf{A} = \text{Đ}_{\sigma_1}(f), \quad \mathbf{B} = \text{Đ}_{\sigma_2}(f)$
$\mathbf{I_B} \circ f_{(\sigma)} \;=\; f_{(\sigma)} \circ \mathbf{I_A}$

e) $\Longrightarrow \; <f> \in_{<\sigma>} \mathcal{F}^*[\,\mathbf{A}, \mathbf{B}\,), \quad \mathbf{A} = \text{Đ}_{\sigma_1}(f) \subseteq \text{Đ}_{\omega_2}(g)$
$<g> \in_{<\omega>} \mathcal{F}_1^*[\,\mathbf{B}, \mathbf{A}\,), \quad \mathbf{B} = \text{Đ}_{\omega_1}(g) \subseteq \text{Đ}_{\sigma_2}(f)$

f) $\Longrightarrow \; h_{(\tau)} \;=\; g_{(\omega)} \circ f_{(\sigma)}, \quad \text{Đ}_{\sigma_2}(f) = \text{Đ}_{\omega_1}(g)$

5.1 Kategories

A *Kategory* is a specific type of tupleset that defines a set of objects and functions between these objects that preserve the behavioral characteristics of a Category.

Definition 5.1 *Komplete $\mathcal{F}$-Space:* $\mathcal{K}^*[\,\mathbf{A}, \mathbf{B}] \;=\; \mathcal{F}^*[\,\mathbf{A}, \mathbf{B}] \;\cup\; \{\, <\mathbf{A}>^{<@\mathbf{A}>},\; <\mathbf{B}>^{<@\mathbf{B}>}\,\}.$

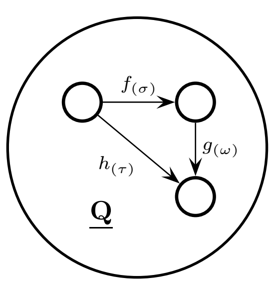
Definition 5.2 *Kategory:* $Kat(\mathbf{Q}) \iff \; Tupset(\mathbf{Q}) = 1 \;\; \&$

$(a)\;\; (\forall \mathbf{f}, \sigma)(\, <\mathbf{f}> \in_{<\sigma>} \mathbf{Q} \;\longrightarrow\; <\mathbf{f}> \in_{<\sigma>} \mathcal{K}^*[\text{Đ}_{\sigma_1}(\mathbf{f}), \text{Đ}_{\sigma_2}(\mathbf{f})]\;\;),$

$(b)\;\; Ob(\mathbf{Q}) \;=\; \{\, <\mathbf{A}>^{<@\mathbf{A}>} \colon (\exists\, \mathbf{x}, \mathbf{s}, \mathbf{B})\big(\mathbf{x} \in_s \mathbf{Q} \;\&\; x \in_s \big(\mathcal{F}^*[\mathbf{A}, \mathbf{B}] \cup \mathcal{F}^*[\mathbf{B}, \mathbf{A}]\big)\big\}$

$(c)\;\; (\forall \mathbf{f}, \sigma, \mathbf{g}, \omega)\big(\, <\mathbf{f}> \in_{<\sigma>} \mathbf{Q} \;\;\&\;\; <\mathbf{g}> \in_{<\omega>} \mathbf{Q} \;\;\&\;\; (\text{Đ}_{\sigma_2}(\mathbf{f}) = \text{Đ}_{\sigma_1}(\mathbf{g})) \;\longrightarrow$
$(\exists\, \mathbf{h}, \tau)(\;\mathbf{h}_{(\tau)} = \mathbf{g}_{(\omega)} \circ \mathbf{f}_{(\sigma)} \;\;\&\;\; <\mathbf{h}> \in_{<\tau>} \mathbf{Q}\,)\big)$

$(d)\;\; (\forall \mathbf{f}, \sigma)\big(\, <\mathbf{f}> \in_{<\sigma>} \mathbf{Q} \;\;\&\;\; \mathbf{A} = \text{Đ}_{\sigma_1}(\mathbf{f}) \;\;\&\;\; \mathbf{B} = \text{Đ}_{\sigma_2}(\mathbf{f}) \;\longrightarrow$
$(\;\mathbf{I_B} \circ \mathbf{f}_{(\sigma)} = \mathbf{f}_{(\sigma)} \circ \mathbf{I_A} \;\;\&\;\; <\mathbf{A}> \in_{<@A>} \mathbf{Q} \;\;\&\;\; <\mathbf{B}> \in_{<@B>} \mathbf{Q}\,)\big).$



$\Longrightarrow$

$$\mathbf{Q} \;=\; \{x^s : \; x \in_s \big(\mathcal{K}^*[\mathbf{A}, \mathbf{B}] \;\cup\; \mathcal{K}^*[\mathbf{B}, \mathbf{C}] \;\cup\; \mathcal{K}^*[\mathbf{A}, \mathbf{C}]\big)\}$$
$$\mathbf{A} \;=\; \text{Đ}_{\sigma_1}(f) = \text{Đ}_{\tau_1}(h)$$
$$\mathbf{B} \;=\; \text{Đ}_{\omega_1}(g) = \text{Đ}_{\sigma_2}(f)$$
$$\mathbf{C} \;=\; \text{Đ}_{\omega_2}(g) = \text{Đ}_{\tau_2}(h)$$
$$Ob(\mathbf{Q}) \;=\; \{\, <\mathbf{A}>^{<@\mathbf{A}>},\; <\mathbf{B}>^{<@\mathbf{B}>},\; <\mathbf{C}>^{<@\mathbf{C}>}\}$$
$$\mathbf{I_C} \circ h_{(\tau)} \circ \mathbf{I_A} \;\;=\;\; \mathbf{I_C} \circ g_{(\omega)} \circ \mathbf{I_B} \circ f_{(\sigma)} \circ \mathbf{I_A}$$
$$Kat(\mathbf{Q})$$

5.2 Functors & Derived Functions

A Functor is defined to be a Tupleset of degree 2 where the members are pairings of composable functions. Thus in XST, a Functor is a binary function whose derived functions behave so as to be commensurate with Category theory.

Definition 5.3 *Functor:* $Fntr(\mathbf{F}) \iff Tupset(\mathbf{F}) = 2 \ \ \& \ \ Kat\big(\text{Ɖ}_{<1>}(\mathbf{F})\big) \ \ \& \ \ Kat\big(\text{Ɖ}_{<2>}(\mathbf{F})\big)$

$$(\forall\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F}\,) \longrightarrow \Big((<\mathbf{f}> \in_{<\sigma>} \mathcal{F}^*[\text{Ɖ}_{\sigma_1}(\mathbf{f}), \text{Ɖ}_{\sigma_2}(\mathbf{f})] \ \ \& \ \ <\mathbf{g}> \in_{<\omega>} \mathcal{F}^*[\text{Ɖ}_{\omega_1}(\mathbf{g}), \text{Ɖ}_{\omega_2}(\mathbf{g})]\,) \ \&$$

$$\Big((\exists\, \mathbf{A},\mathbf{B})(<\mathbf{A},\mathbf{B}> \in_{<@\mathbf{A},\,@\mathbf{B}>} \mathbf{F}\,) \ \& \ \big((\,\mathbf{A} = \text{Ɖ}_{\sigma_1}(\mathbf{f}) \ \& \ \mathbf{B} = \text{Ɖ}_{\omega_1}(\mathbf{g})\,) \ or \ (\,\mathbf{A} = \text{Ɖ}_{\sigma_2}(\mathbf{f}) \ \& \ \mathbf{B} = \text{Ɖ}_{\omega_2}(\mathbf{g})\,)\big)\Big)\Big) \ \&$$

$$\Big(\{<\mathbf{f},\mathbf{x}>^{<a,s>}, <\mathbf{g},\mathbf{y}>^{<b,t>}, <\mathbf{h},\mathbf{z}>^{<c,u>}\} \subseteq \mathbf{F} \ \ \& \ \ \mathbf{h}_{(c)} = \mathbf{g}_{(b)} \circ \mathbf{f}_{(a)} \ \longrightarrow \ \mathbf{z}_{(u)} = \mathbf{y}_{(t)} \circ \mathbf{x}_{(s)}\Big).$$

Definition 5.4 *Derived Functor Functions:*

(a) $SS(\mathbf{F}) = \{<\mathbf{x}>^{<\mathbf{y}>} : \ (\exists\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F} \ \ \& \ \ <\mathbf{x}> \in_{<\mathbf{y}>} \mathcal{F}^*[\text{Ɖ}_{\sigma_1}(\mathbf{f}), \text{Ɖ}_{\omega_1}(\mathbf{g})]\,\}.$

(b) $TT(\mathbf{F}) = \{<\mathbf{x}>^{<\mathbf{y}>} : \ (\exists\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F} \ \ \& \ \ <\mathbf{x}> \in_{<\mathbf{y}>} \mathcal{F}^*[\text{Ɖ}_{\sigma_2}(\mathbf{f}), \text{Ɖ}_{\omega_2}(\mathbf{g})]\,\}.$

(c) $ST(\mathbf{F}) = \{<\mathbf{x}>^{<\mathbf{y}>} : \ (\exists\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F} \ \ \& \ \ <\mathbf{x}> \in_{<\mathbf{y}>} \mathcal{F}^*[\text{Ɖ}_{\sigma_1}(\mathbf{f}), \text{Ɖ}_{\omega_2}(\mathbf{g})]\,\}.$

(d) $TS(\mathbf{F}) = \{<\mathbf{x}>^{<\mathbf{y}>} : \ (\exists\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F} \ \ \& \ \ <\mathbf{x}> \in_{<\mathbf{y}>} \mathcal{F}^*[\text{Ɖ}_{\sigma_2}(\mathbf{f}), \text{Ɖ}_{\omega_1}(\mathbf{g})]\,\}.$

(e) $SS^{(-)}(\mathbf{F}) = \{<\mathbf{x}>^{<\mathbf{y}>} : \ (\exists\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F} \ \ \& \ \ <\mathbf{x}> \in_{<\mathbf{y}>} \mathcal{F}^*[\text{Ɖ}_{\omega_1}(\mathbf{g}), \text{Ɖ}_{\sigma_1}(\mathbf{f})]\,\}.$

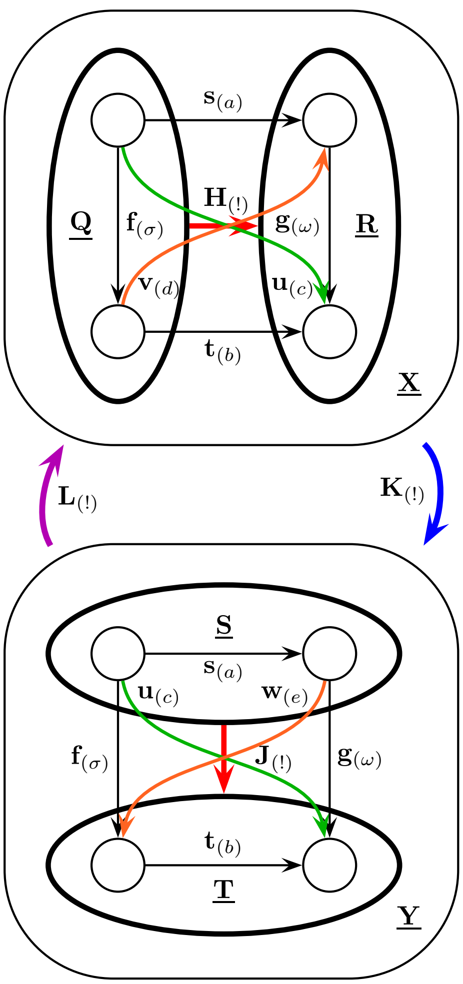
(f) $TT^{(-)}(\mathbf{F}) = \{<\mathbf{x}>^{<\mathbf{y}>} : \ (\exists\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F} \ \ \& \ \ <\mathbf{x}> \in_{<\mathbf{y}>} \mathcal{F}^*[\text{Ɖ}_{\omega_2}(\mathbf{g}), \text{Ɖ}_{\sigma_2}(\mathbf{f})]\,\}.$

(g) $ST^{(-)}(\mathbf{F}) = \{<\mathbf{x}>^{<\mathbf{y}>} : \ (\exists\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F} \ \ \& \ \ <\mathbf{x}> \in_{<\mathbf{y}>} \mathcal{F}^*[\text{Ɖ}_{\omega_1}(\mathbf{g}), \text{Ɖ}_{\sigma_2}(\mathbf{f})]\,\}.$

(h) $TS^{(-)}(\mathbf{F}) = \{<\mathbf{x}>^{<\mathbf{y}>} : \ (\exists\, \mathbf{f}, \sigma, \mathbf{g}, \omega)(<\mathbf{f},\mathbf{g}> \in_{<\sigma,\,\omega>} \mathbf{F} \ \ \& \ \ <\mathbf{x}> \in_{<\mathbf{y}>} \mathcal{F}^*[\text{Ɖ}_{\omega_2}(\mathbf{g}), \text{Ɖ}_{\sigma_1}(\mathbf{f})]\,\}.$

(i) $DF(\mathbf{F}) \ = \ SS(\mathbf{F}) \cup TT(\mathbf{F}) \cup ST(\mathbf{F}) \cup TS(\mathbf{F}).$

(j) $DF^{(-)}(\mathbf{F}) \ = \ SS^{(-)}(\mathbf{F}) \cup TT^{(-)}(\mathbf{F}) \cup ST^{(-)}(\mathbf{F}) \cup TS^{(-)}(\mathbf{F}).$



$\Longrightarrow$

$\mathbf{Q} \ = \ \{x^u : \ x \in_u \mathcal{K}^*[\,\text{Ɖ}_{\sigma_1}(\mathbf{f}), \text{Ɖ}_{\sigma_2}(\mathbf{f})\,]\}$

$\mathbf{R} \ = \ \{y^v : \ y \in_v \mathcal{K}^*[\,\text{Ɖ}_{\omega_1}(\mathbf{g}), \text{Ɖ}_{\omega_2}(\mathbf{g})\,]\}$

$<\mathbf{H}> \in_{<!>} \mathcal{F}^*[\,\mathbf{Q},\mathbf{R}\,] \ \ \& \ \ Fntr(\mathbf{H}) \ \ \& \ \ \mathbf{H} \subseteq \mathbf{Q}\times\mathbf{R}$

$DF(\mathbf{H}) = \{<\mathbf{s}>^{<a>}, \ <\mathbf{t}>^{<b>}, \ <\mathbf{u}>^{<c>}, \ <\mathbf{v}>^{<d>}\}$

$\mathbf{g}_{(\omega)} \circ \mathbf{s}_{(a)} \ = \ \mathbf{g}_{(\omega)} \circ \mathbf{v}_{(d)} \circ \mathbf{f}_{(\sigma)} \ = \ \mathbf{t}_{(b)} \circ \mathbf{f}_{(\sigma)} \ = \ \mathbf{u}_{(c)}$

$Kat(\mathbf{Q}), \ \ Kat(\mathbf{R}), \ \ Kat(\mathbf{X})$

$<\mathbf{K}> \in_{<!>} \mathcal{F}^*[\,\mathbf{X},\mathbf{Y}\,] \ \ \& \ \ Fntr(\mathbf{K}) \ \ \& \ \ \mathbf{K} \subseteq \mathbf{X}\times\mathbf{Y}$

$<\mathbf{L}> \in_{<!>} \mathcal{F}^*[\,\mathbf{Y},\mathbf{X}\,] \ \ \& \ \ Fntr(\mathbf{L}) \ \ \& \ \ \mathbf{L} \subseteq \mathbf{Y}\times\mathbf{X}$

$\Longrightarrow$

$\mathbf{S} \ = \ \{x^u : \ x \in_u \mathcal{K}^*[\,\text{Ɖ}_{a_1}(\mathbf{s}), \text{Ɖ}_{a_2}(\mathbf{s})\,]\}$

$\mathbf{T} \ = \ \{y^v : \ y \in_v \mathcal{K}^*[\,\text{Ɖ}_{b_1}(\mathbf{t}), \text{Ɖ}_{b_2}(\mathbf{t})\,]\}$

$<\mathbf{J}> \in_{<!>} \mathcal{F}^*[\,\mathbf{S},\mathbf{T}\,] \ \ \& \ \ Fntr(\mathbf{J}) \ \ \& \ \ \mathbf{J} \subseteq \mathbf{S}\times\mathbf{T}$

$DF(\mathbf{J}) = \{<\mathbf{f}>^{<\sigma>}, \ <\mathbf{g}>^{<\omega>}, \ <\mathbf{u}>^{<c>}, \ <\mathbf{w}>^{<e>}\}$

$\mathbf{t}_{(b)} \circ \mathbf{f}_{(\sigma)} \ = \ \mathbf{t}_{(b)} \circ \mathbf{w}_{(e)} \circ \mathbf{s}_{(a)} \ = \ \mathbf{g}_{(\omega)} \circ \mathbf{s}_{(a)} \ = \ \mathbf{u}_{(c)}$

$Kat(\mathbf{S}), \ \ Kat(\mathbf{T}), \ \ Kat(\mathbf{Y})$

5.3 More Machinary

Additional operations and definitions are required to support Category theory capabilities in terms of XST. For example, in Category theory, $Fntr(\mathbf{F}) \implies \mathbf{F}(\,\mathbf{g} \circ \mathbf{f}\,) \;=\; \mathbf{F}(\mathbf{g}) \circ \mathbf{F}(\mathbf{f})$. To support this condition in XST requires a definition for $\mathbf{F}_{(\sigma)}(\mathbf{g}_{(\omega)})$.

Definition 5.5 *Function Application:* $\mathbf{F}_{(\sigma)}(\mathbf{g}_{(\omega)}) \;=\; \mathbf{h}_{(\tau)} \iff (\exists\,\mathbf{j},\mathbf{k})(\,\mathbf{k}_{(!\omega!)} \;=\; (\mathbf{g}_{(\omega)})^{*}) \;\;\&$

$$(\,\mathbf{F}\big[\{<\mathbf{k}>^{<!\omega!>}\}\big]_{\sigma} \;=\; \{<\mathbf{j}>^{<!\tau!>}\})\;\&\;(\,\mathbf{j}_{(!\tau!)} \;=\; (\mathbf{h}_{(\tau)})^{*}\,)$$

Note: $!\omega! \;=<\mathsf{N}(tup(\omega_1)), \mathsf{N}(tup(\omega_2), tup(\omega_1))>.$

Assertion 5.1 $Fntr(\mathbf{F}) \implies \mathbf{F}_{(!)}(\,\mathbf{g}_{(\omega)} \circ \mathbf{f}_{(\sigma)}\,) \;=\; \mathbf{F}_{(!)}(\mathbf{g}_{(\omega)}) \circ \mathbf{F}_{(!)}(\mathbf{f}_{(\sigma)}).$

Assertion 5.2 $\mathbf{F}_{(\sigma)}(\mathbf{g}_{(\omega)}) = \mathbf{h}_{(\tau)} \;\;\&\;\; \mathbf{g}_{(\omega)} = \mathbf{f}_{(\alpha)} \;\;\&\;\; \mathbf{F}_{(\sigma)}(\mathbf{f}_{(\alpha)}) = \mathbf{k}_{(\mu)} \implies \mathbf{h}_{(\tau)} = \mathbf{k}_{(\mu)}.$

Definition 5.6 *Domain Product:* $<\mathbf{f}_{(\sigma)}, \mathbf{g}_{(\omega)}> = \mathbf{h}_{(!\sigma!\sigma\omega)}$ *where*

$$\mathbf{h} \;=\; \{z^{v} : (\exists x, s, y, t)(\,x \in_{s} \mathbf{f} \;\;\&\;\; y \in_{t} \mathbf{g}) \;\&\; (\,x^{/\sigma_1/} = y^{/\omega_1/} \neq \emptyset \;\;\&\;\; s^{/\sigma_1/} = t^{/\omega_1/} \neq \emptyset) \;\;\&$$

$$(\,z \;=\; x^{/\sigma_1/} \circ x^{/\sigma_2/} \circ y^{/\omega_2/} \neq \emptyset \;\;\&\;\; v \;=\; s^{/\sigma_1/} \circ s^{/\sigma_2/} \circ t^{/\omega_2/})\,\},\;\; with$$

$$!\sigma!\sigma\omega \;=\; <\;\mathsf{N}(tup(\sigma_1)),\;\; \mathsf{N}(tup(\sigma_2) + tup(\omega_2), tup(\sigma_1))\;>.$$

Assertion 5.3 $<\mathbf{f}_{(\sigma)}, \mathbf{g}_{(\omega)}, \mathbf{h}_{(\tau)}> \;=\; <\mathbf{f}_{(\sigma)}, <\mathbf{g}_{(\omega)}, \mathbf{h}_{(\tau)}>> \;=\; <<\mathbf{f}_{(\sigma)}, \mathbf{g}_{(\omega)}>, \mathbf{h}_{(\tau)}>.$

Definition 5.7 *Relative Product:* $\mathbf{f}\big/_{\sigma}^{\omega}\,\mathbf{g} = \{\,z^{s} :\; (tup(\sigma,\omega) = 2) \;\;\&\;\; (Tupset(\mathbf{f},\mathbf{g})) \;\;\&$

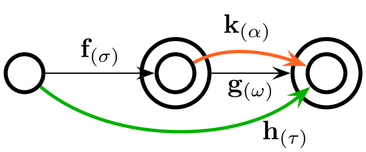
$$(\exists x, a, y, b)(<x> \in_{<a>} \mathbf{f} \;\;\&\;\; <y> \in_{<b>} \mathbf{g}) \;\;\&$$

$$(x^{/\sigma_2/} = y^{/\omega_1/} \neq \emptyset \;\;\&\;\; a^{/\sigma_2/} = b^{/\omega_1/} \neq \emptyset) \;\;\&$$

$$(z = x^{/\sigma_1/} \cdot y^{/\omega_2/} \neq \emptyset \;\;\&\;\; s = a^{/\sigma_1/} \cdot b^{/\omega_2/} \neq \emptyset)\,\}.$$

Note: $!\sigma! \;=<\mathsf{N}(tup(\sigma_1)), \mathsf{N}(tup(\sigma_2), tup(\sigma_1))>.$

Conjecture 5.1 $\mathbf{h}_{(\tau)} \;=\; \mathbf{g}_{(\omega)} \circ \mathbf{f}_{(\sigma)} \implies \mathbf{h}_{(\tau)} = \big(\mathbf{f}\big/_{\sigma}^{\omega}\,\mathbf{g}\big)_{(!\sigma\omega)},\;\; !\sigma\omega \;=<\mathsf{N}(tup(\sigma_1)), \mathsf{N}(tup(\omega_2), tup(\sigma_1))>.$



$$\implies \quad \mathbf{h}_{(\tau)} \;?\!=\!?\; \big(\mathbf{f}\big/_{\sigma}^{\omega}\,\mathbf{g}\big)_{(!\sigma\omega)},\quad \mathbf{h}_{(\tau)} \;=\; \mathbf{k}_{(\alpha)} \circ \mathbf{f}_{(\sigma)},\quad \mathbf{g}_{(\omega)} \circ \mathbf{f}_{(\sigma)} \;=?$$

Definition 5.8 *Domain Projected Function:* $\Phi(\mathbf{T}) = \Big\{<\mathbf{A},\mathbf{t}>^{<@\mathbf{A},\eta>} : (\exists\,\{<\mathbf{f},\mathbf{g},\mathbf{h}>^{<\sigma,\omega,\tau>}\} \subseteq \mathbf{T})(\exists\,\mathbf{x}, a, \mathbf{y}, b) \;\;\&$

$$\Big[\,1)\;\Big(\mathbf{A} = \mathbf{Đ}_{\sigma_1}(\mathbf{f}),\;\; <\mathbf{x}> \in_{<a>} \mathcal{F}^{*}[\mathbf{A}, \mathbf{Đ}_{\omega_1}(\mathbf{g})],\;\; <\mathbf{y}> \in_{<b>} \mathcal{F}^{*}[\mathbf{A}, \mathbf{Đ}_{\tau_1}(\mathbf{h})],\;\; <\mathbf{t}> \in_{<\eta>} \mathcal{F}^{*}[\mathbf{Đ}_{\omega_1}(\mathbf{g}), \mathbf{Đ}_{\tau_1}(\mathbf{h})]\,\Big)$$

or

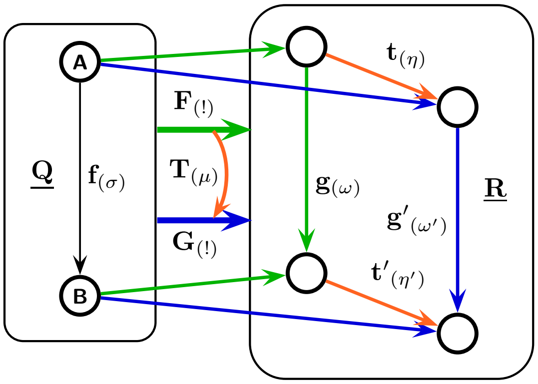
$$2)\;\Big(\mathbf{A} = \mathbf{Đ}_{\sigma_2}(\mathbf{f}),\;\; <\mathbf{x}> \in_{<a>} \mathcal{F}^{*}[\mathbf{A}, \mathbf{Đ}_{\omega_2}(\mathbf{g})],\;\; <\mathbf{y}> \in_{<b>} \mathcal{F}^{*}[\mathbf{A}, \mathbf{Đ}_{\tau_2}(\mathbf{h})],\;\; <\mathbf{t}> \in_{<\eta>} \mathcal{F}^{*}[\mathbf{Đ}_{\omega_2}(\mathbf{g}), \mathbf{Đ}_{\tau_2}(\mathbf{h})]\,\Big)\Big]$$

$$\&$$

$$\Big(\,\mathbf{y}_{(b)} \;=\; \mathbf{t}_{(\eta)} \circ \mathbf{x}_{(a)}\,\Big)\Big\}$$

Convention 5.1 $\Phi(\mathbf{T})_{(!)}(\mathbf{A}) \;=\; \Phi(\mathbf{T})_{(!)}(\mathbf{A}_{(@\mathbf{A})}).$

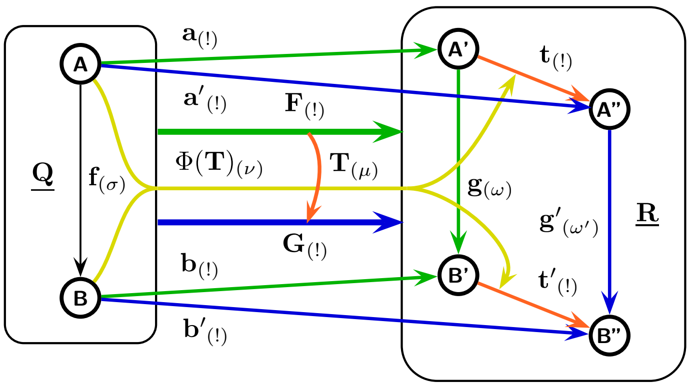
5.4 Natural Transformations



$\Longrightarrow$

$$Kat(\mathbf{Q}),\ \ Kat(\mathbf{R}),\ \ Fntr(\mathbf{F}),\ \ Fntr(\mathbf{G})$$
$$<\mathbf{F}> \in_{<!>} \mathcal{F}^*[\,\mathbf{Q},\mathbf{R}\,],\quad <\mathbf{G}> \in_{<!>} \mathcal{F}^*[\,\mathbf{Q},\mathbf{R}\,]$$
$$\text{Đ}_{<1>}(\mathbf{F}) \ = \ \text{Đ}_{<1>}(\mathbf{G})$$
$$\mathbf{T} = \mathbf{F} \Big/^{!}_{!} \mathbf{G},\ \ \Phi(\mathbf{T})_{(!)}(\mathbf{A}) = \mathbf{t}_{(\eta)},\ \ \Phi(\mathbf{T})_{(!)}(\mathbf{B}) = \mathbf{t}'_{(\eta')}$$
$$<\mathbf{T}> \in_{<\nu>} \mathcal{F}^*[\,\mathbf{Q},\ \mathbf{R}\times\mathbf{R}\,],\ \ \nu = <<1>,<2,3>>$$
$$<\mathbf{T}> \in_{<\mu>} \mathcal{F}^*[\,\mathbf{F},\ \mathbf{G}\,],\ \ \mu = <<1,2>,<1,3>>$$
$$\mathbf{T}_{(\mu^*)}\big(\mathbf{F}_{(!)}(x)\big) = \mathbf{G}_{(!)}(x),\ \ \mu^* = <<2>,<3>>$$
$$\Phi(\mathbf{T})_{(\nu)}(\mathbf{B})_{(\eta')} \circ \mathbf{F}_{(!)}(\mathbf{f}_{(\sigma)}) \ = \ \mathbf{G}_{(!)}(\mathbf{f}_{(\sigma)}) \circ \Phi(\mathbf{T})_{(\nu)}(\mathbf{A})_{(\eta)}$$
$$\mathbf{t}'_{(\eta')} \circ \mathbf{g}_{(\omega)} \ = \ \mathbf{g}'_{(\omega')} \circ \mathbf{t}_{(\eta)}$$

Following is a detailed example of a mapping, $\mathbf{f}_{(\sigma)}$, of **A** to **B** with a list reversal defined by $\mathbf{F}_{(!)},\ \mathbf{G}_{(!)}\ \&\ \mathbf{T}_{(\mu)}$.



$\Longrightarrow$

$$\Phi(\mathbf{T}) \in_{<\nu>} \mathcal{F}^*[\,Ob(\mathbf{Q}),\ \mathbf{R}'\times\mathbf{R}'\,)$$
$$\mathbf{R}' \ = \ \mathbf{R} \sim Ob(\mathbf{R})$$
$$\nu = <<1>,<2,3>>$$
$$\mathbf{a}'_{(!)} \ = \ \mathbf{t}_{(!)} \circ \mathbf{a}_{(!)}$$
$$\mathbf{b}'_{(!)} \ = \ \mathbf{t}'_{(!)} \circ \mathbf{b}_{(!)}$$

1) $\mathsf{A} \ = \ \{ <a>^{<\emptyset>}, <b>^{<\emptyset>}, <c>^{<\emptyset>} \},\quad \mathsf{B} \ = \ \{ <\mathbf{f}_{(\sigma)}(a)>^{<\emptyset>}, <\mathbf{f}_{(\sigma)}(b)>^{<\emptyset>}, <\mathbf{f}_{(\sigma)}(c)>^{<\emptyset>} \}$

2) $\mathsf{A'} \ = \ \{ <a>^{<1>}, <b>^{<2>}, <c>^{<3>} \},\quad \mathsf{B'} \ = \ \{ <\mathbf{f}_{(\sigma)}(a)>^{<1>}, <\mathbf{f}_{(\sigma)}(b)>^{<2>}, <\mathbf{f}_{(\sigma)}(c)>^{<3>} \}$

3) $\mathsf{A''} \ = \ \{ <a>^{<3>}, <b>^{<2>}, <c>^{<1>} \},\quad \mathsf{B''} \ = \ \{ <\mathbf{f}_{(\sigma)}(a)>^{<3>}, <\mathbf{f}_{(\sigma)}(b)>^{<2>}, <\mathbf{f}_{(\sigma)}(c)>^{<1>} \}$

4) $\mathbf{f} \ = \ \{ <a, \mathbf{f}_{(\sigma)}(a)>^{<\emptyset,\emptyset>}, <b, \mathbf{f}_{(\sigma)}(b)>^{<\emptyset,\emptyset>}, <c, \mathbf{f}_{(\sigma)}(c)>^{<\emptyset,\emptyset>} \}$

5) $\mathbf{g} \ = \ \{ <a, \mathbf{f}_{(\sigma)}(a)>^{<1,1>}, <b, \mathbf{f}_{(\sigma)}(b)>^{<2,2>}, <c, \mathbf{f}_{(\sigma)}(c)>^{<3,3>} \},\quad \omega = <<1>,<2>>$

6) $\mathbf{g}' \ = \ \{ <a, \mathbf{f}_{(\sigma)}(a)>^{<3,3>}, <b, \mathbf{f}_{(\sigma)}(b)>^{<2,2>}, <c, \mathbf{f}_{(\sigma)}(c)>^{<1,1>} \},\quad \omega' = <<1>,<2>>$

7) $\mathbf{F} \ = \ \big\{ <\mathbf{f}, \mathbf{g}>^{<\sigma,\omega>}, <\mathsf{A}, \mathsf{A'}>^{<@\mathsf{A},@\mathsf{A'}>}, <\mathsf{B}, \mathsf{B'}>^{<@\mathsf{B},@\mathsf{B'}>} \big\}$

8) $\mathbf{G} \ = \ \big\{ <\mathbf{f}, \mathbf{g}'>^{<\sigma,\omega'>}, <\mathsf{A}, \mathsf{A''}>^{<@\mathsf{A},@\mathsf{A''}>}, <\mathsf{B}, \mathsf{B''}>^{<@\mathsf{B},@\mathsf{B''}>} \big\}$

9) $\mathbf{T} \ = \ \big\{ <\mathbf{f}, \mathbf{g}, \mathbf{g}'>^{<\sigma,\omega,\omega'>}, <\mathsf{A}, \mathsf{A'}, \mathsf{A''}>^{<@\mathsf{A},@\mathsf{A'},@\mathsf{A''}>}, <\mathsf{B}, \mathsf{B'}, \mathsf{B''}>^{<@\mathsf{B},@\mathsf{B'},@\mathsf{B''}>} \big\}$

10) $\Phi(\mathbf{T}) = \big\{ <\mathsf{A}, a, a>^{<@\mathsf{A},1,3>},\ <\mathsf{A}, b, b>^{<@\mathsf{A},2,2>},\ <\mathsf{A}, c, c>^{<@\mathsf{A},3,1>},$
$<\mathsf{B}, \mathbf{f}_{(\sigma)}(a), \mathbf{f}_{(\sigma)}(a)>^{<@\mathsf{B},1,3>}, <\mathsf{B}, \mathbf{f}_{(\sigma)}(b), \mathbf{f}_{(\sigma)}(b)>^{<@\mathsf{B},2,2>}, <\mathsf{B}, \mathbf{f}_{(\sigma)}(c), \mathbf{f}_{(\sigma)}(c)>^{<@\mathsf{B},3,1>} \big\}$

11) $\mathbf{t} \ = \ \Phi(\mathbf{T})_{(\nu)}(\mathsf{A}) \ = \ \big\{ <a, a>^{<1,3>},\ <b, b>^{<2,2>},\ <c, c>^{<3,1>} \big\}$

12) $\mathbf{t}' \ = \ \Phi(\mathbf{T})_{(\nu)}(\mathsf{B}) \ = \ \big\{ <\mathbf{f}_{(\sigma)}(a), \mathbf{f}_{(\sigma)}(a)>^{<1,3>}, <\mathbf{f}_{(\sigma)}(b), \mathbf{f}_{(\sigma)}(b)>^{<2,2>}, <\mathbf{f}_{(\sigma)}(c), \mathbf{f}_{(\sigma)}(c)>^{<3,1>} \big\}$

13) $\mathbf{a} \ = \ \{ <a, a>^{<\emptyset,1>}, <b, b>^{<\emptyset,2>}, <c, c>^{<\emptyset,3>} \}$

14) $\mathbf{a}' \ = \ \{ <a, a>^{<\emptyset,3>}, <b, b>^{<\emptyset,2>}, <c, c>^{<\emptyset,1>} \}$

15) $\mathbf{b} \ = \ \{ <\mathbf{f}_{(\sigma)}(a), \mathbf{f}_{(\sigma)}(a)>^{<\emptyset,1>}, <\mathbf{f}_{(\sigma)}(b), \mathbf{f}_{(\sigma)}(b)>^{<\emptyset,2>}, <\mathbf{f}_{(\sigma)}(c), \mathbf{f}_{(\sigma)}(c)>^{<\emptyset,3>} \}$

16) $\mathbf{b}' \ = \ \{ <\mathbf{f}_{(\sigma)}(a), \mathbf{f}_{(\sigma)}(a)>^{<\emptyset,3>}, <\mathbf{f}_{(\sigma)}(b), \mathbf{f}_{(\sigma)}(b)>^{<\emptyset,2>}, <\mathbf{f}_{(\sigma)}(c), \mathbf{f}_{(\sigma)}(c)>^{<\emptyset,1>} \}$

6 CONJECTURES & QUESTIONS

Since Kategories and Functors are defined as sets, under what conditions do set operations on Kategories and Functors produce Kategories or Functors? Some conditions seem conspicuously obvious, others decptively obvious, some highly contrived, while many legitimate constructions probably will not occur except to those highly familiar with Category theory.

Definition 6.1 *Power Kategory:* $PK(\mathbf{Q}) = \{ <\mathbf{R}>^{<@\mathbf{R}>} \colon \mathbf{R} \subseteq \mathbf{Q} \ \& \ Kat(\mathbf{R})\}$.

Definition 6.2 *Power Functor:* $PF(\mathbf{F}) = \{ <\mathbf{G}>^{<@\mathbf{G}>} \colon \mathbf{G} \subseteq \mathbf{F} \ \& \ Fntr(\mathbf{G})\}$.

[XST_ETC5a: 12/04/05]